AC Resistance

DETERMINE THE AC RESISTANCE IN A CIRCUIT WITH INDUCTIVE AND RESISTIVE LOADS.

- Determine the amplitude and phase of the overall resistance as a function of frequency for a series circuit.
- Determine the amplitude and phase of the overall resistance as a function of frequency for a parallel circuit.

BASIC PRINCIPLES

In AC circuits, it is common to use complex numbers to describe the resistance in circuits with inductors because this actually makes calculation easier. This is because not only the amplitude of the current and voltage is a factor, but also the phase relationships between the two need to be taken into account (this complex resistance is usually called impedance). Series and parallel circuits with both ohmic and inductive resistance can then be described quite easily, although in each case, only the real component is measurable).

The complex resistance (impedance) of a coil of inductance \( L \) in a circuit with an alternating current of frequency \( f \) is as follows:

\[
X = i \cdot X_0 = i \cdot \omega \cdot L
\]

Angular frequency \( \omega = 2 \pi \cdot f \).

Therefore the total resistance (impedance) of a series circuit containing a coil with a resistance \( R \) is

\[
Z_s = i \cdot 2 \cdot \pi \cdot f \cdot L + R.
\]

For a parallel circuit, the total resistance can be assigned thus:

\[
Z_p = \frac{1}{1 + \frac{1}{i \cdot 2 \cdot \pi \cdot f \cdot L + R}}
\]

The usual way of writing this is as follows:

\[
Z = Z_0 \cdot \exp(i \cdot \phi).
\]

This becomes

\[
Z_p = Z_{0p} \cdot \exp(i \cdot \phi_p) = \frac{1}{\sqrt{(2 \cdot \pi \cdot f \cdot L)^2 + R^2}} \cdot \exp(i \cdot \phi_p)
\]

where \( \tan \phi_p = \frac{2 \cdot \pi \cdot f \cdot L}{R} \)

and

\[
Z_s = Z_{0s} \cdot \exp(i \cdot \phi_s) = \frac{1}{\sqrt{(2 \cdot \pi \cdot f \cdot L)^2 + R^2}} \cdot \exp(i \cdot \phi_s)
\]

where \( \tan \phi_s = \frac{R}{2 \cdot \pi \cdot f \cdot L} \).
If for each of the total impedance values \( Z = Z_s \) and \( Z_p \) the following voltage is applied:

\[
(7) \quad U = U_0 \cdot \exp(i \cdot 2 \cdot \pi \cdot f \cdot t)
\]

this will result in the following current flowing:

\[
(8) \quad I = \frac{U_0}{Z_0} \cdot \exp(i \cdot (2 \cdot \pi \cdot f \cdot t - \varphi)) = I_0 \cdot \exp(i \cdot (2 \cdot \pi \cdot f \cdot t - \varphi))
\]

In this experiment this will be measured by finding the drop in voltage \( U_m(t) \) across a load resistor \( R_m \) (Fig. 2, 3), which is of such a value that \( U_m \ll U_i \), i.e. nearly all of the voltage drops across \( Z_s \) or \( Z_p \). The current obtained in this way flows through \( Z_s \) and through \( Z_p \) since both these resistors are in series with \( R_m \) (see equivalent circuit diagram in Fig. 2, 3). Since \( U_m(t) = I(t) \cdot R_m \), the change in voltage over time \( U_m(t) \) will also be reflected in the change in current over time \( I(t) \).

### LIST OF EQUIPMENT

1. Plug-In Board for Components 1012902 (U33250)
2. Resistor 1 \( \Omega \), 2 W, P2W19 1012903 (U333011)
3. Resistor 100 \( \Omega \), 2 W, P2W19 1012910 (U333018)
4. Function Generator FG 100 @230V 1009957 (U8533600-230)
   or @115V 1009956 (U8533600-115)
5. PC Oscilloscope 2x25 MHz 1020857 (U11830)
6. HF Patch Cord, BNC/4 mm Plug 1002748 (U11257)
7. Set of 15 Experiment Leads, 75 cm 1 mm² 1002840 (U13800)
8. Coil S with 800 Taps 1001001 (U8498080)
9. Coil S with 1200 Taps 1001002 (U8498085)

### SET UP AND PROCEDURE

**Series circuit**

- Assemble a set-up for measurement of the series circuit (as in Fig. 1, left) in accordance with the sketch of the circuit diagram (Fig. 2) in which \( R_m = 10 \Omega \), \( C = 1 \mu F \) and the coil has \( N = 1200 \) turns and inductance \( L = 23.0 \) mH.

- Connect the output signal \( U_m(t) = I(t) \cdot R_m \) to channel 1 (CH1) of the oscilloscope and the input signal \( U(t) \) to channel 2 (CH2).

- Configure the following starting parameters for the PC oscilloscope:

  - **Time/DIV**: 25 \( \mu s \)
  - **Volts/DIV CH1**: 2.00 mV AC
  - **Volts/DIV CH2**: 2.00 V AC
  - **Trigger Mode**: Auto
  - **Sweep**: Edge
  - **Source**: CH2
  - **Slope**: +
  - **Level**: 0.00 \( \mu V \)
Note
The parameters Time/DIV und Volts/DIV for CH1 will need to be modified during the course of this sequence of measurements.

- Select Sine as the signal form for the function generator and set the amplitude of the input signal to \( U_0 = 6 \text{ V} \). Adjust the amplitude knob in such a way that the maximum or minimum of the sine-wave signal on channel CH2 of the oscilloscope (with 2 V per division) coincides with exactly 3 divisions.
- Set up frequencies of 10000 Hz, 5000 Hz, 2000 Hz, 1000 Hz, 500 Hz, 200 Hz and 50 Hz in sequence on the function generator. Calculate the corresponding periods of oscillation by means of the equation \( T = 1 / f \) and enter the results into Table 1 along with the frequencies.
- Read off the amplitude \( U_{\text{io}} \) of the output signal \( U_{\text{in}}(t) \) from the oscilloscope and enter the results into Table 1.

- Read off the time differences \( \Delta t \) between zero crossings of the signals \( U(t) \) and \( U_{\text{in}}(t) \) from the oscilloscope and enter the values into Table 1.
- Repeat the measurements for all the same frequencies using the coil with \( N = 800 \) turns and inductance \( L = 10.4 \text{ mH} \). Enter these values into Table 1.

Parallel circuit

- Assemble a set-up for measurement of the parallel circuit (as in Fig. 1) in accordance with the sketch of the circuit diagram (Fig. 3) in which \( R_0 = 10 \Omega \), \( C = 1 \text{ \mu F} \) and the coil has \( N = 1200 \) turns and inductance \( L = 23.0 \text{ mH} \).
- Carry out the measurements in the same way as for the series circuit. Use the same initial parameters for the USB oscilloscope but set Volts/DIV for CH1 to 20.0 \text{ V AC}
- Enter all the measurement values into Table 2.

SAMPLE MEASUREMENT

Tab. 1: Measurement values for series circuit.

<table>
<thead>
<tr>
<th>( f / \text{Hz} )</th>
<th>( T / \text{ms} )</th>
<th>( U_{\text{io}} / \text{mV} )</th>
<th>( \Delta t / \mu\text{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 1200 )</td>
<td>( N = 800 )</td>
<td>( N = 1200 )</td>
<td>( N = 800 )</td>
</tr>
<tr>
<td>( L = 23.0 \text{ mH} )</td>
<td>( L = 10.4 \text{ mH} )</td>
<td>( L = 23.0 \text{ mH} )</td>
<td>( L = 10.4 \text{ mH} )</td>
</tr>
<tr>
<td>10000</td>
<td>0.1</td>
<td>4.1</td>
<td>7.8</td>
</tr>
<tr>
<td>5000</td>
<td>0.2</td>
<td>7.8</td>
<td>14.9</td>
</tr>
<tr>
<td>2000</td>
<td>0.5</td>
<td>18.7</td>
<td>31.6</td>
</tr>
<tr>
<td>1000</td>
<td>1.0</td>
<td>32.1</td>
<td>43.3</td>
</tr>
<tr>
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<td>2.0</td>
<td>41.8</td>
<td>49.3</td>
</tr>
<tr>
<td>200</td>
<td>5.0</td>
<td>47.7</td>
<td>51.7</td>
</tr>
<tr>
<td>50</td>
<td>20.0</td>
<td>49.5</td>
<td>52.3</td>
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</table>

Tab. 2: Measurement values for parallel circuit.

<table>
<thead>
<tr>
<th>( f / \text{Hz} )</th>
<th>( T / \text{ms} )</th>
<th>( U_{\text{io}} / \text{mV} )</th>
<th>( \Delta t / \mu\text{s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 1200 )</td>
<td>( N = 800 )</td>
<td>( N = 1200 )</td>
<td>( N = 800 )</td>
</tr>
<tr>
<td>( L = 23.0 \text{ mH} )</td>
<td>( L = 10.4 \text{ mH} )</td>
<td>( L = 23.0 \text{ mH} )</td>
<td>( L = 10.4 \text{ mH} )</td>
</tr>
<tr>
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<td>61.7</td>
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<tr>
<td>5000</td>
<td>0.2</td>
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<td>62.6</td>
</tr>
<tr>
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<td>63.0</td>
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<td>1.0</td>
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<td>50</td>
<td>20.0</td>
<td>323.6</td>
<td>431.5</td>
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</table>
EVALUATION

- Calculate the sizes of the inductive impedance using the formula \( X_L = 2 \cdot \pi \cdot f \cdot L \) (see equation 1) and enter the results into Table 3.
- From the values of \( U_{R0} \) (tables 1, 2) and \( R_m \) (1 \( \Omega \)), calculate the current amplitudes using the equation \( I_0 = \frac{U_{R0}}{R_m} \) and enter the results into Table 3.
- Calculate the total impedance values \( Z_{0S} \) and \( Z_{0P} \) using the equation \( Z_0 = U_0 / I_0 \) \( (U_0 = 6 \text{ V}) \) and enter the results into Table 3.
- From the values for the period of oscillation \( T \) and the time difference \( \Delta t \) (tables 1, 2) calculate the phase shifts using the equation \( \varphi = 360^\circ \cdot \Delta t / T \) and enter the results into Table 3.
- Plot the total impedance values \( Z_{0S} \) and \( Z_{0P} \) and phase shifts \( \varphi \) for the series and parallel circuits as a function of \( X_L \) (Fig. 4 – 7).

- Calculate the theoretical total impedance values \( Z_{0S} \) and \( Z_{0P} \) and phase shifts \( \varphi \) according to equation (5) for the series circuit and equation (6) for the parallel circuit.

\[
(9) \quad Z_{0S} = \sqrt{R^2 + X_L^2}, \quad \varphi = \arctan \left( \frac{X_L}{R} \right)
\]

\[
(10) \quad Z_{0P} = \frac{1}{\sqrt{R^2 + X_L^2}}, \quad \varphi = \arctan \left( \frac{R}{X_L} \right)
\]

Draw them as continuous lines into Figs. 4 – 7.

Conclusion

If the inductive impedance is large, the resistance of the series circuit will have the value of the inductive resistance and the parallel circuit will have the value of the ohmic resistance. The phase shift is between 0° and 90° and equals 45° if the ohmic and inductive resistance values are the same.

Tab. 3: Calculation of absolute value \( X_L \) of inductive impedance from measurements of current amplitude \( I_0 \), total impedance values \( Z_{0S} \) and \( Z_{0P} \) and phase shift \( \varphi \) for series and parallel circuits.

<table>
<thead>
<tr>
<th>( N ) ( L ) / mH</th>
<th>Series circuit</th>
<th>Parallel circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_L / \Omega )</td>
<td>( I_0 / mA )</td>
<td>( Z_{0S} / \Omega )</td>
</tr>
<tr>
<td>1200 23.0</td>
<td>1445.1</td>
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<td>722.6</td>
<td>7.8</td>
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<tr>
<td></td>
<td>3.3</td>
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</table>
Fig. 4: Total impedance of series circuit for $N = 800$ (●) and $N = 1200$ (○) turns.

Fig. 5: Phase shift in series circuit for $N = 800$ (●) and $N = 1200$ (○) turns.

Fig. 6: Total impedance of parallel circuit for $N = 800$ (●) and $N = 1200$ (○) turns.

Fig. 7: Phase shift in series circuit for $N = 800$ (●) and $N = 1200$ (○) turns.