

Voltage on a Parallel-Plate Capacitor

STATIC MEASUREMENT OF VOLTAGE AS A FUNCTION OF PLATE SPACING

- Static measurement of voltage on a parallel plate capacitor as a function of plate spacing.
- Confirmation of the proportionality between voltage and plate spacing for small plate spacings.

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Fig. 1: Experimental setup

PRINCIPLES

The charged plates of a parallel-plate capacitor exert an attractive force on each other. Therefore, mechanical work must be performed to increase the plate spacing of a charged parallel-plate capacitor that has been disconnected from all supply leads. The energy supplied to the capacitor in this way can be verified by observing an increase in the voltage between the plates, provided that no current can flow between the plates during the voltage measurement.

Ideal parallel-plate capacitor

For a more precise description of the relationships, consider the homogeneous electric field E between the capacitor plates carrying the electric charges Q and $-Q$. The following applies:

$$(1) \quad E = \frac{1}{\epsilon_0} \cdot \frac{Q}{A}$$

A : plate area,

$$\epsilon_0 = 8,85 \cdot 10^{-12} \frac{\text{V} \cdot \text{s}}{\text{A} \cdot \text{m}} : \text{vacuum permittivity.}$$

If no current can flow when the plate spacing d is changed, the electric charge Q , and therefore the electric field E , remains unchanged.

The following applies to the electric charge:

$$(2) \quad Q = C \cdot U$$

The capacitance of a parallel-plate capacitor is given by:

$$(3) \quad C = \epsilon_0 \cdot \frac{A}{d}$$

d : plate spacing.

For small plate spacings, where the electric field may be regarded as homogeneous, the following relationship therefore applies between the voltage U across the capacitor and the electric field E :

$$(4) \quad U = E \cdot d$$

That is, the voltage U is proportional to the plate spacing d .

This relationship is verified experimentally using the electric field meter as an electrostatic voltmeter. This ensures that no current can flow through the voltmeter between the capacitor plates and that the electric charge Q on the capacitor plates remains unchanged.

Influence of the measuring equipment

At the beginning of the measurement, the parallel-plate capacitor is charged once by applying the voltage $U(d_0)$ at a plate spacing d_0 . In addition to the capacitance of the parallel-plate capacitor, the capacitance C_M of the measurement setup must be taken into account.

Since the fixed capacitor plate is connected to the upper side of the electric field mill of the electric field meter, while the movable capacitor plate is connected to the lower side of the electric field mill and grounded, the entire arrangement must be regarded as a parallel circuit of capacitances. In particular, the capacitances of the parallel-plate capacitor and the measurement setup add up to give the total capacitance:

$$(5) \quad C_{\text{tot}} = C(d_0) + C_M$$

where

$$(6) \quad C(d_0) = \epsilon_0 \cdot \frac{A}{d_0}$$

The total charge is therefore:

$$(7) \quad Q_{\text{tot}} = [C(d_0) + C_M] \cdot U(d_0)$$

If the plate spacing is increased to $d > d_0$, the total charge remains constant, provided that the parallel-plate capacitor remains electrically isolated after charging:

$$(8) \quad Q_{\text{tot}} = [C(d) + C_M] \cdot U(d)$$

From Eqs. (7) and (8), it follows that

$$(9) \quad Q_{\text{tot}} = [C(d) + C_M] \cdot U(d) = [C(d_0) + C_M] \cdot U(d_0) \Leftrightarrow U(d) = U(d_0) \cdot \frac{C(d_0) + C_M}{C(d) + C_M}$$

In particular, if the capacitance C_M is negligible, that is, $C_M \ll C(d_0)$, $C(d)$, the following is obtained:

$$(10) \quad U(d) = U(d_0) \cdot \frac{C(d_0)}{C(d)}$$

To eliminate the influence of the capacitance C_M from the measured voltage, the measured voltage must be multiplied by

a correction factor that relates the voltages in Eqs. (9) and (10). The corrected voltage is therefore:

$$(11) \quad U_{\text{corr}} = U_{\text{meas}} \cdot \frac{U(d_0) \cdot \frac{C(d_0)}{C(d)}}{U(d_0) \cdot \frac{C(d_0) + C_M}{C(d) + C_M}} = U_{\text{meas}} \cdot \frac{C(d) + C_M}{C(d)} \cdot \frac{C(d_0)}{C(d_0) + C_M}$$

U_{meas} : measured voltage,
 U_{corr} : corrected voltage.

Note:

At $d = d_0$, $U_{\text{corr}} = U_{\text{meas}}$ because $C(d) = C(d_0)$.

EQUIPMENT

1 Electric Field Meter	1021405
or	
1 Electric Field Meter (115 V, 50/60 Hz)	1021406
1 Parallel-Plate Capacitor, Type D	1006798
1 DC Power Supply, 0–20 V, 0–5 A	1003312
or	
1 DC Power Supply, 0–20 V, 0–5 A (115 V, 50/60 Hz)	1003311
1 Digital Multimeter P 1035	1002781
3 Connecting Leads, Blue, Red, 2 pcs.	1024540
1 Connecting Leads, Black, 2pcs.	1002850

SETUP

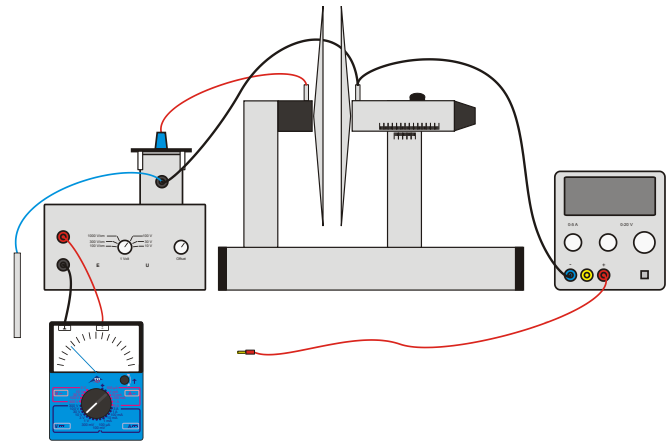


Fig. 2: Experimental setup

- Set up the experiment as shown in Fig. 1 or Fig. 2.
- Place the voltage-measuring plate, measuring range 1× on the shielding cylinder of the electric field meter so that the distance from the shielding plate is as small as possible. Secure the voltage-measuring plate using the knurled screw.
- Connect the fixed capacitor plate to the voltage measuring plate.
- Connect the movable capacitor plate to the ground jack on the shielding cylinder of the electric field meter and to the negative terminal of the power supply.
- Connect the grounding rod to the ground jack on the shielding cylinder of the electric field meter.

- Connect one end of a connecting lead to the positive terminal of the power supply. Place the other, free end of the lead on the table and leave it unconnected.
- Connect the multimeter to the voltage output of the electric field meter for voltage measurement.
- Set the range selector switch of the electric field meter to 100 V, switch on the electric field meter, and wait approximately 3 minutes for it to stabilize.

PROCEDURE

- Use the fine adjustment screw of the parallel-plate capacitor to set the plate spacing to $d = 2 \text{ mm} = d_0$.
- Switch on the power supply and set the voltage to $U(d_0) = 10 \text{ V}$.
- To discharge the parallel-plate capacitor, touch the fixed capacitor plate with the grounding rod, thereby short-circuiting the plates. At the same time, use the offset control to adjust the zero point on the electric field meter.
- Remove the grounding rod from the fixed capacitor plate and hold it in your hand for equipotential bonding.
- Keep the grounding rod in your hand throughout the entire measurement.
- To charge the parallel-plate capacitor, touch the fixed capacitor plate with the free end of the cable connected to the positive terminal of the power supply.
- Once the parallel-plate capacitor is charged, remove the cable from the fixed capacitor plate again so that the parallel-plate capacitor is disconnected from the positive terminal.
- Read the measured voltage U_{meas} on the multimeter for the set plate spacing $d = 2 \text{ mm} = d_0$ and enter the value in Table 1.
- Increase the plate spacing in 2 mm increments up to $d = 18 \text{ mm}$, proceeding without unnecessary delay. At each step, read the measured voltage U_{meas} for the set plate spacing and enter the value in Table 1.

MEASUREMENT EXAMPLE

Table 1: Measured voltage U_{meas} and corrected voltage U_{corr} as a function of plate spacing d

d / mm	$U_{\text{meas}} / \text{V}$	$U_{\text{corr}} / \text{V}$
2	10.0	10.0
4	16.2	19.8
6	20.7	29.8
8	24.2	40.2
10	26.9	50.6
12	28.9	60.7
14	30.1	69.9
16	31.5	80.1
18	32.0	88.4

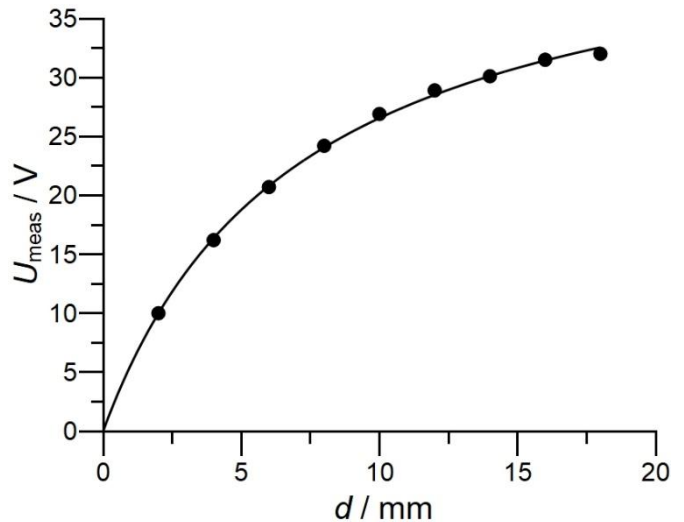


Fig. 3: Measured voltage U_{meas} as a function of plate spacing d , with a curve fitted according to Eq. (9) to determine the capacitance C_M

ANALYSIS

- Plot the measured voltages U_{meas} from Table 1 against the set plate spacings d (Fig. 3).

The measuring points do not exhibit the linear relationship expected according to Eq. (4). To correct for the influence of the measurement setup according to Eq. (11), its capacitance C_M must be determined.

- Fit Eq. (9) to the measuring points $U_{\text{meas}}(d)$ to determine the unknown capacitance C_M .

Parameters:

$$d_0 = 2 \text{ mm}$$

$$U(d_0) = 10 \text{ V}$$

$$A = 0,05 \text{ m}^2: \text{ plate area}$$

$$C(d_0) = 2,21 \cdot 10^{-10} \text{ F (from Eq. (6))}$$

$$C(d) = 4,43 \cdot 10^{-13} \text{ F} / d \text{ (from Eq. (3))}$$

The fit yields $C_M = 62,5 \pm 0,6 \text{ pF}$.

- Using Eqs. (3), (6), and (11), as well as the value of C_M determined from the fit, calculate the corrected voltages U_{corr} from the measured voltages U_{meas} in Table 1 and enter the results in Table 1.
- Plot the corrected voltages U_{corr} from Table 1 against the set plate spacings d and fit a straight line through the origin to the data (Fig. 4).

The linear relationship $U \propto d$ underlying the ideal parallel-plate capacitor is revealed.

Notes:

The plot corrected in this way for the systematic influence of the measurement setup illustrates the linear relationship $U \propto d$, but does not independently verify it. The known electric field $E = U_0 / d_0 = 5 \text{ V/mm}$ is reproduced by the slope of the fitted straight line. For independent verification, the capacitance C_M must be measured separately.

Within the range of plate spacings considered, fringing effects caused by the increasing inhomogeneity of the electric field are not significantly visible compared with the dominant influence of the capacitance of the measurement setup, because the plate diameter (approximately 25 cm) is large relative to the set plate spacings (2 – 18 mm).

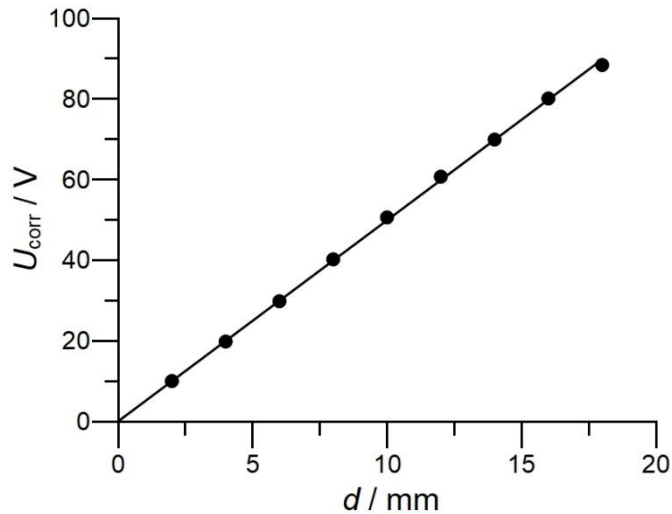


Fig. 4: Corrected voltage U_{corr} as a function of plate spacing d