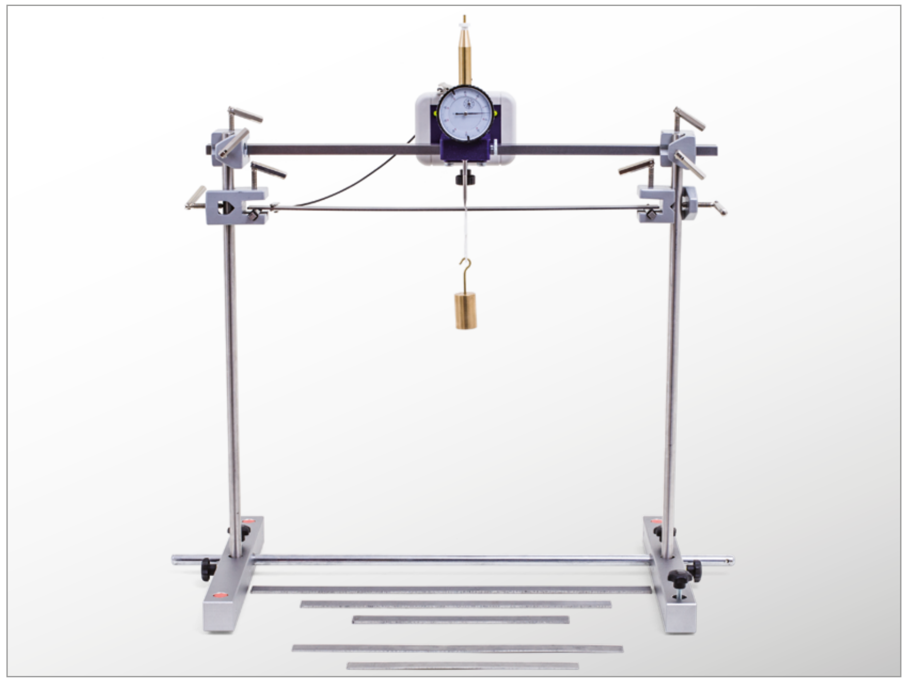
UE1090200 | BENDING OF FLAT BEAMS



OBJECTIVE

Measurement of deformation of flat beams supported at both ends and determination of modulus of elasticity

SUMMARY

A flat, level beam's resistance to deformation in the form of bending by an external force can be calculated mathematically if the degree of deformation is much smaller than the length of the beam. The deformation is proportional to the modulus of elasticity E of the material from which the beam is made. In this experiment, the deformation due to a known force is measured and the results are used to determine the modulus of elasticity for both steel and aluminium.

EXPERIMENT PROCEDURE

- Measure the deformation profile with loads in the center and loads away from the center.
- Measure the deformation as a function of the force.
- Measure the deformation as a function of the length, width and breadth as well as how it depends on the material and determine the modulus of elasticity of the materials.

REQUIRED APPARATUS

Quantity	Description	Item Number
1	Apparatus for Measuring Young's Modulus	1018527
1	Young's Modulus Supplementary Set	1018528
1	Pocket Measuring Tape, 2 m	1002603
1	External Micrometer	1002600

BASIC PRINCIPLES

A flat, level beam's resistance to deformation in the form of bending by an external force can be calculated mathematically if the degree of deformation is much smaller than the length of the beam. The deformation is proportional to the modulus of elasticity E of the material from which the beam is made. Therefore the deformation due to a known force can be measured and the results are used to determine the modulus of elasticity.

For the calculation, the beam is sliced into parallel segments which are compressed on the inside by the bending and stretched on the outside. Neutral segments undergo no compression or extension. The relative extension or compression ε of the other threads and the associated tension σ depends on their distance z from the neutral segments:

(1) $$\varepsilon(z) = \frac{s+\Delta s(z)}{s} = \frac{z}{\rho(x)} \quad \text{and} \quad \sigma(z) = E\cdot\varepsilon(z)$$

$\rho(x)$: Local radius of curvature due to bending

The curvature therefore involves the local bending moment:

(2) $$M(x) = \int_A \sigma(z)\cdot z\cdot \mathrm{d}A = \frac{1}{\rho(x)}\cdot E\cdot I$$

where $I = \int_A z^2\cdot \mathrm{d}A$: Area moment of inertia

As an alternative to the radius of curvature $\rho(x)$, in this experiment the deformation profile $w(x)$, by which the neutral segments are shifted from their rest position, will be measured. This can be calculated as follows, as long as the changes $\mathrm{d}w(x)/\mathrm{d}x$ due to the deformation are sufficiently small:

(3) $$\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}(x) = \frac{1}{\rho(x)} = \frac{M(x)}{E\cdot I}$$

the deformation profile is obtained from this by double integration.

A typical example is to observe a beam of length L, which is supported at both ends and to which a downward force F acts at a point a. In a state of equilibrium the sum of all the forces acting is zero:

(4) $$F_1 + F_2 - F = 0$$

Similarly, the sum of all the moments acting on the beam at an arbitrary point x is also zero:

(5) $$M(x) - F_1\cdot x - F_2\cdot(L-x) + F\cdot(a-x) = 0$$

No curvature or deformation arises at the ends of the beam, i.e. $M(0) = M(L) = 0$ and $w(0) = w(L) = 0$. This means that $M(x)$ is fully determinable:

(6) $$M(\zeta) = \begin{cases} F\cdot L\cdot(1-\alpha)\cdot\zeta; & 0\le\zeta\le\alpha \\ F\cdot L\cdot\alpha\cdot(1-\zeta); & \alpha<\zeta\le 1 \end{cases}$$

$$\text{where } \zeta = \frac{x}{L} \text{ and } \alpha = \frac{a}{L}$$

The deformation profile is obtained by double integration

(7) $$w(\zeta) = \begin{cases} \frac{F\cdot L^3}{E\cdot I}\cdot\left[(1-\alpha)\cdot\frac{\zeta^3}{6} - \left(\frac{\alpha^3}{6} - \frac{\alpha^2}{2} - \frac{\alpha}{3}\right)\cdot\zeta\right] \\ \frac{F\cdot L^3}{E\cdot I}\cdot\left[\frac{\alpha^3}{6} - \left(\frac{\alpha^3}{6} + \frac{\alpha}{3}\right)\zeta + \frac{\alpha}{2}\cdot\zeta^2 - \frac{\alpha}{6}\zeta^3\right] \end{cases}$$

In the experiment the shape of this profile is checked for load at the center of the beam (α = 0.5) and off-center (α < 0.5).

EVALUATION

When the load is in the center, then $w(x=\frac{L}{2}, a=\frac{L}{2}) = -\frac{F\cdot L^3}{48\cdot E\cdot I}$.

For a rectangle of width b and height d, the following calculation is made:

$$I = \int_A z^2\cdot \mathrm{d}A = \int_{-\frac{d}{2}}^{\frac{d}{2}} z^2\cdot b\cdot \mathrm{d}z = \frac{d^3}{12}\cdot b$$

Then $w(x=\frac{L}{2}, a=\frac{L}{2}) = -\frac{1}{4}\cdot\frac{F}{E}\cdot\frac{L^3}{d^3}\cdot\frac{1}{b}$.

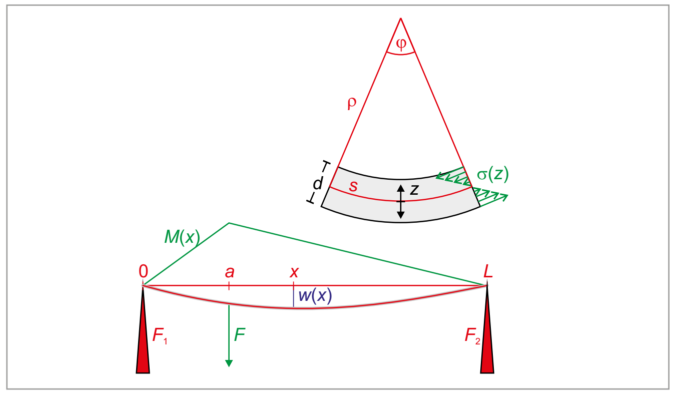


Fig. 1: Sketch of the deformation profile.

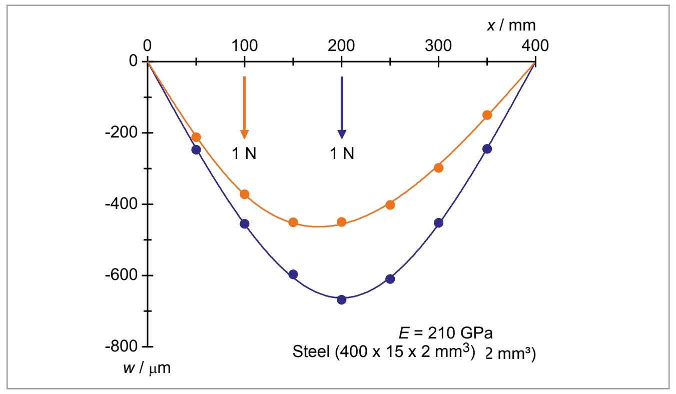


Fig. 2: Measured and calculated deformation profile for load acting at center and off-center

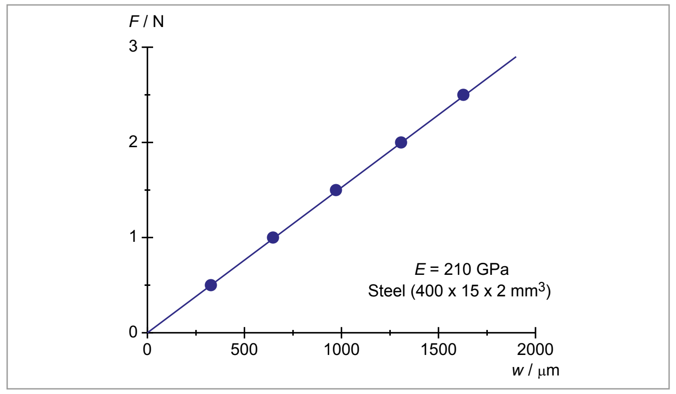


Fig. 3: Confirmation of Hooke's law

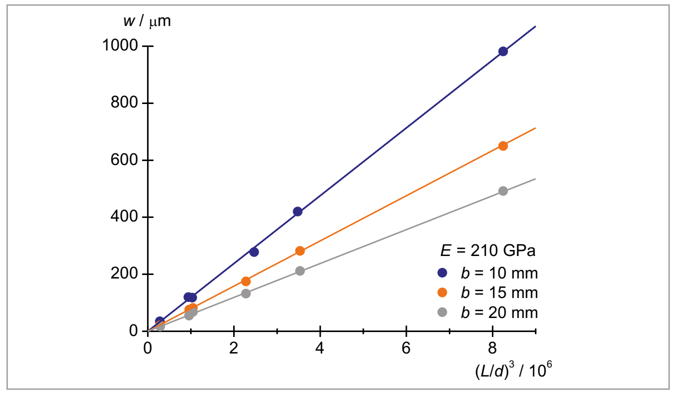


Fig. 4: How the deformation depends on $(L/d)^3$

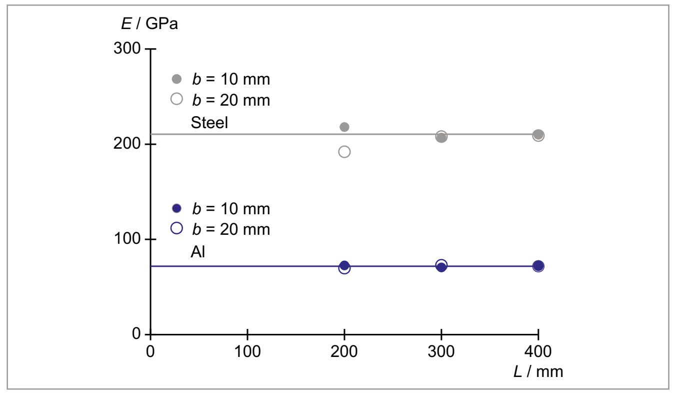


Fig. 5: Modulus of elasticity of steel and aluminium