Mechanics

Acoustics

Sound propagation in solids

TO DETERMINE THE SPEED OF SOUND PROPAGATED BY LONGITUDINAL AND TRANSVERSE WAVES IN SOLIDS.

- To determine the speed of sound for longitudinal waves in polyacrylic from the propagation time of a 1-MHz ultra-sound signal.
- To measure the transmission of longitudinal and transverse sound waves in solids through an inclined, plane-parallel plate.
- To determine the speed of sound for longitudinal and transverse waves from the critical angle of total reflection.
- To determine the elastic modulus *E*, the shear modulus *G* and Poisson's ratio of a solid μ from the two speeds.

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BASIC PRINCIPLES

In gases and liquids, sound is propagated exclusively in the form of longitudinal waves. In the process, the sound pressure oscillates around an equilibrium value and generates oscillating regions of compression and rarefaction. Sound also penetrates solids in the form of transverse waves in which the shear stress oscillates. Transverse waves can propagate through solids because solids possess the necessary shear force required for conducting sound.

Longitudinal and transverse waves possess different speeds which depend on the density ρ and the elastic constant of the solid. The speed of longitudinal waves, given by

$$(1)\quad c_L = \sqrt{\frac{E}{\rho}\cdot\frac{1-\mu}{(1+\mu)\cdot(1-2\mu)}}$$

E: where *E* is the elastic modulus and μ Poisson's ratio,

is greater than that of transverse waves

$$(2)\quad c_T = \sqrt{\frac{G}{\rho}}$$

where *G* is the shear modulus

The relation between the elastic modulus *E*, shear modulus *G* of a solid and Poisson's ratio is given by the following equation:

$$(3)\quad \frac{E}{G} = 2\cdot(1+\mu)$$

It is therefore possible to calculate all three magnitudes of elasticity, given that the two sound speeds c_L and c_T are known.

In the experiment, first measure the propagation time *t* of a 1-MHz ultrasound signal through three polyacrylic cylinders of different lengths *s*. Plot the values in an *s*-*t* graph (see fig. 1). From the inclination of the best-fit line through the measured values, we get the longitudinal sound speed in polyacrylic.

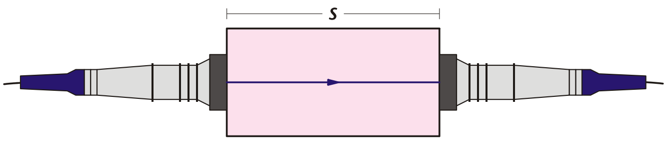

Fig. 1: Experiment set-up to measure the time for an ultra-sound signal to pass through a solid body of length *s*

Subsequently, fill a trough with water and place it in the path of the wave. Measure the transit time. The transit time is reduced by placing a thin plane-parallel plate made of polyacrylic or aluminium in the path of the wave. This is due to the fact that sound propagates faster in the plate material than in water. Take accurate readings behind the water trough for the two distinct ultrasound signals caused due to the different propagation times for longitudinal and transversal sound waves in solids (see fig. 2).

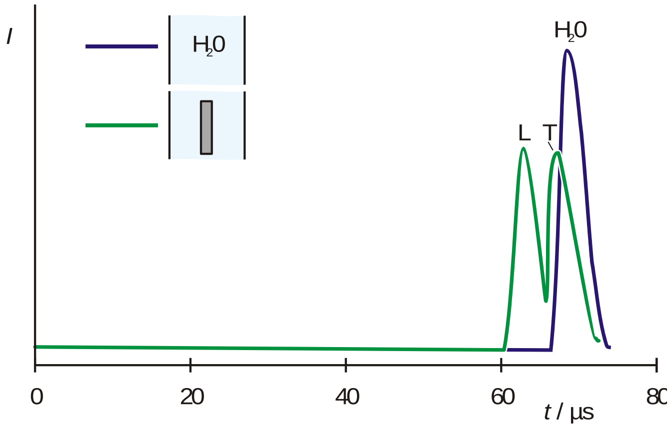

Fig. 2: Ultrasound signal after penetrating a water trough (blue: without plane-parallel plate, green: with plane-parallel plate)

LIST OF APPARATUS

1 Ultrasonic Echoscope GS200	1018616 (U100102)
2 Ultrasonic Probe 1 MHz GS200	1018618 U10015
1 Equipment Set "Ultrasound in Solids"	1002584 (U10020)
1 Aluminium Test Block	1002585 (U10022)
1 Set of 3 Cylinders	1002588 (U10026)
1 Ultrasonic Coupling Gel	1002588 (XP999)

Also needed:
1 PC with Microsoft Windows

If the plate is inclined at an angle α to the incident wave, then, according to Snell's law, the wave is refracted and the two refracted waves are at angles β_L and β_T (see fig. 3).

$$(4)\quad \frac{c}{\sin\alpha} = \frac{c_L}{\sin\beta_L} = \frac{c_T}{\sin\beta_T}$$

c: where *c* is the speed of sound in water

As the two sound speeds c_L and c_T through the solid are greater than the speed of sound *c* in water, we can eventually observe the phenomenon of total reflection – distinctly for longitudinal and transverse waves – in which the transmitted signals fully disappear. The corresponding speeds can be measured from the critical angles α_L for longitudinal waves and α_T for transverse waves:

$$(5)\quad c_L = \frac{c}{\sin\alpha_L} \text{ and } c_T = \frac{c}{\sin\alpha_T}$$

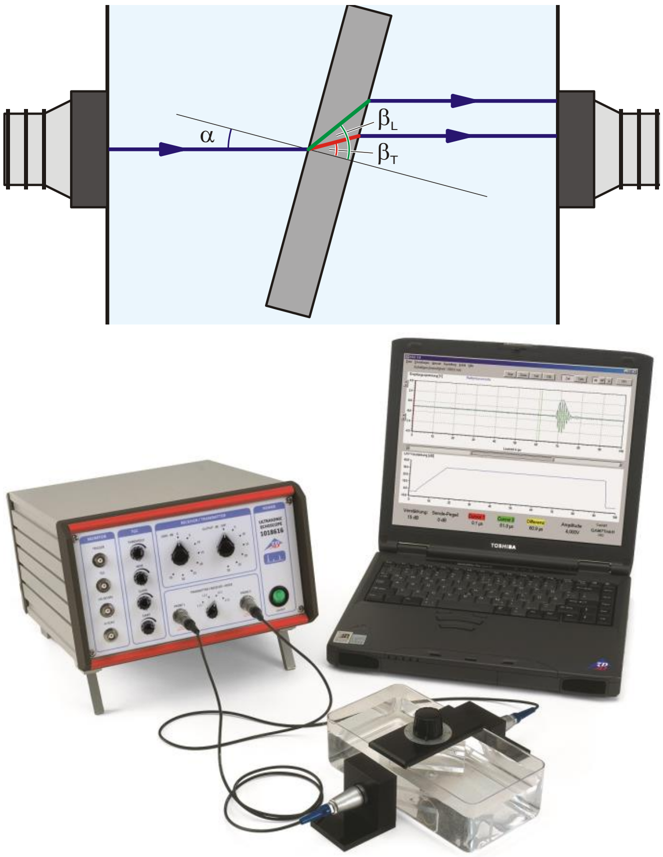

Fig. 3: Experimental set-up for determining the speed of sound for longitudinal and transverse waves from the critical angles of total reflection.

SET-UP

- Connect the ultrasonic echoscope to the PC.
- If necessary, install the data-processing program on the PC.
- Insert the two ultrasonic probes into their holders.
- Connect the first probe to the PROBE 1 output of the ultrasonic echoscope and the second probe to the PROBE 2 output.

EXPERIMENT PROCEDURE

- Switch on the ultrasonic echoscope and start the data-processing program.

a) Measure transit times of longitudinal waves:

- Smear the front surfaces of the ultrasonic probes with a thick layer of ultrasonic coupling gel and press them firmly against the end surfaces of the long polyacrylate cylinder.
- On the ultrasonic echoscope set the controls OUTPUT dB and GAIN dB and the parameters THRESHOLD, WIDE and SLOPE to give as large a transit time signal as possible but without overloading.
- Measure the transit time *t* from the beginning of the transmitter pulse to the beginning of the receiver pulse and enter the value in Table 1.
- Make similar measurements with the medium and short polyacrylate cylinders and enter the transit times in Table 1.

b) Compare longitudinal and transverse waves:

- Replace the polyacrylate cylinder by the sound trough and press the ultrasonic probes firmly against its two long sides.
- Fill the trough with water.
- On the ultrasonic echoscope set the controls OUTPUT dB and GAIN dB and the parameters THRESHOLD, WIDE and SLOPE to give as large a transit time signal as possible but without overloading.
- On the echoscope screen use the cursor to mark the beginning of the receiver pulse.
- Set the aluminium plate with its sample holder perpendicular to the sound propagation direction and observe the splitting and shifting of the transit time signal.
- Turn the aluminium plate and determine the angle of rotation α_L that causes the longitudinal wave signal (the one on the left) to disappear.
- Continue to turn the aluminium plate and determine the angle of rotation α_T that causes the transverse wave signal (the one on the right) to disappear.
- Replace the aluminium plate and sample holder by the polyacrylate plate with sample holder and set it perpendicular to the sound propagation direction.
- Turn the polyacrylate plate and determine the angle of rotation α_L that causes the longitudinal wave signal (the one on the left) to disappear.
- Continue to turn the polyacrylate plate and determine the angle of rotation α_T that causes the transverse wave signal (the one on the right) to disappear.

SAMPLE MEASUREMENTS AND EVALUATION

a) Measurement of transit times:

Table 1: Transit times *t* in polyacrylate cylinders of length *s*

s / mm	*t* / µs
40	15.7
80	30.6
120	45.3

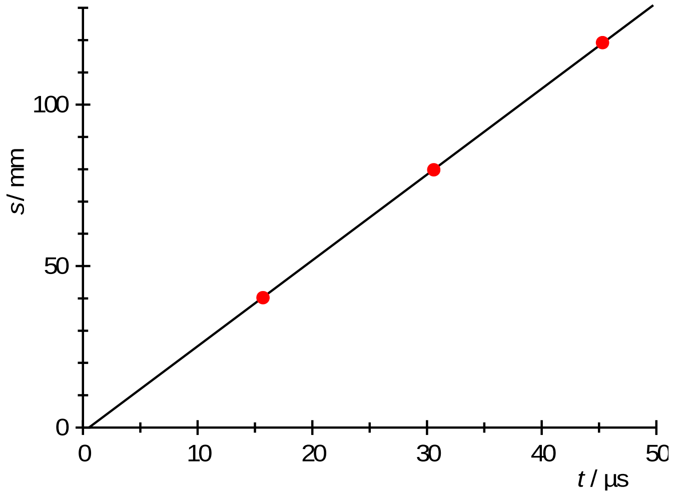

Fig. 4: *s-t* graph of an ultrasound signal in polyacrylic

In the *s-t* plot of the data from Table 1 (see Fig. 4) the line through the points does not pass through the origin, because the measured transit time includes the contributions of the protective film and coupling gel layer on the ultrasonic probes. However, the velocity of sound for longitudinal waves can be determined from the gradient of the straight line. The result is:

$$v_\mathrm{L} = 2660\,\frac{\mathrm{m}}{\mathrm{s}}$$

b) Comparing longitudinal and transverse waves:

Table 2: Critical angles $\square_L$ and $\square_T$ for total reflection of longitudinal and transverse sound waves

	Polyacryl	Aluminium
α_L	33°	14°
sin α_L	0.54	0.24
v_L / m/s	2700	6100
α_T	86°	29°
sin α_T	0.998	0.48
v_T / m/s	1500	3100
μ	0.29	0.33
G / MPa	2700	25000
E / MPa	6900	67000
ρ / g/cm³	1.2	2.7

In Table 2 the velocities of longitudinal and transverse sound waves in polyacrylate and aluminium are calculated from the critical angles for total reflection using Equation 5. The velocity of sound in water that was inserted into the equation is:

$$c = 1485\,\frac{\mathrm{m}}{\mathrm{s}}$$

From equations 1 to 3, we get the characteristic equation for Poisson's ratio µ

$$\mu = \frac{\frac{1}{2}\cdot\left(\frac{c_\mathrm{L}}{c_\mathrm{T}}\right)^2 - 1}{\left(\frac{c_\mathrm{L}}{c_\mathrm{T}}\right)^2 - 1}$$

If the densities ρ are known, the elastic constants can be calculated.