Mechanics

Oscillations

PHYSICS EXPERIMENT

Coupled oscillations

RECORD AND EVALUATE OSCILLATION OF TWO IDENTICAL COUPLED PENDULUMS.

- Record the oscillations when they are in phase and determine the period T_+.
- Record the oscillations when they are out of phase and determine the period T_-.
- Record the oscillations of a coupled pendulum at the maximum beat amplitude and determine the period T of the oscillations and the period T_Δ of the beats.
- Compare the measurements for the two periods with the values calculated from the intrinsic periods T_- and T_+.
- Determine the spring constant of the spring coupling the two pendulums.

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BASIC PRINCIPLES

For oscillation of two coupled pendulums, the oscillation energy is transferred from one pendulum to the other and back again. If the two pendulums are identical and the oscillation is started from a position where one is suspended in its rest position while the other is at a point of maximum deflection, then all the energy in the system is transferred between the pendulums. I.e. one pendulum always comes to rest while the other is swinging at its maximum amplitude. The time between two such occurrences of rest for one pendulum or, more generally, the time between any two instances of minimum amplitude is referred to as the beat period. T_Δ.

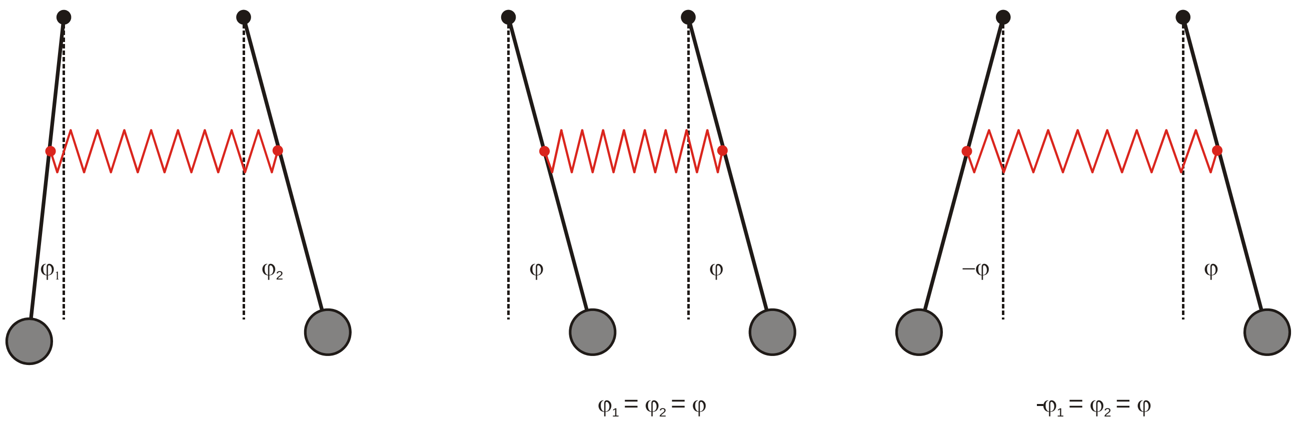
The oscillation of two identical coupled ideal pendulums can be regarded as a superimposition of two natural oscillations. These natural oscillations can be observed when the both pendulums are fully in phase or fully out of phase. In the first case, both pendulums vibrate at the frequency that they would if the coupling to the other pendulum were not present at all. In the second case, the effect of the coupling is at a maximum and the inherent frequency is greater. All other oscillations can be described by superimposing these two natural oscillations.

The equation of motion for the pendulums (for small angles of deflection φ_1 and φ_2) takes the form:

$$\begin{aligned} L\cdot\ddot{\varphi}_1 + g\cdot\varphi_1 + k\cdot(\varphi_1-\varphi_2) &= 0 \\ L\cdot\ddot{\varphi}_2 + g\cdot\varphi_2 + k\cdot(\varphi_2-\varphi_1) &= 0 \end{aligned} \tag{1}$$

g: Acceleration due to gravity, L: length of pendulum, k: coupling constant

Fig. 1: Left: general coupled oscillation, Middle: coupled oscillation in phase, Right: coupled oscillation out of phase



For the motions $\varphi_+ = \varphi_1 + \varphi_2$ and $\varphi_- = \varphi_1 - \varphi_2$ (initially chosen arbitrarily) the equation of motion is as follows:

$$\begin{aligned} &L \cdot \ddot{\varphi}_+ + g \cdot \varphi_+ = 0 \\ &L \cdot \ddot{\varphi}_- + (g + 2k) \cdot \varphi_- = 0 \end{aligned} \tag{2}$$

The solutions

$$\begin{aligned} &\varphi_+ = a_+ \cos(\omega_+ t) + b_+ \sin(\omega_+ t) \\ &\varphi_- = a_- \cos(\omega_- t) + b_- \sin(\omega_- t) \end{aligned} \tag{3}$$

give rise to angular frequencies

$$\omega_+ = \sqrt{\frac{g}{L}} \text{ und } \omega_- = \sqrt{\frac{g+2k}{L}} \tag{4}$$

corresponding to the natural frequencies for in phase or out of phase motion ($\varphi_+ = 0$ for out of phase motion and $\varphi_- = 0$ for in-phase motion).

The deflection of the pendulums can be calculated from the sum or the difference of the two motions, leading to the solutions

$$\begin{aligned} &\varphi_1 = \frac{1}{2}\left(a_+ \cos(\omega_+ t) + b_+ \sin(\omega_+ t) + a_- \cos(\omega_- t) + b_- \sin(\omega_- t)\right) \\ &\varphi_2 = \frac{1}{2}\left(a_+ \cos(\omega_+ t) + b_+ \sin(\omega_+ t) - a_- \cos(\omega_- t) - b_- \sin(\omega_- t)\right) \end{aligned} \tag{5}$$

Parameters a_+, a_-, b_+ and b_- are arbitrary coefficients that can be calculated from the initial conditions for the two pendulums at time $t = 0$.

The easiest case to interpret is where pendulum 1 is deflected by an angle φ_0 from its rest position and released at time 0 while pendulum 2 remains in its rest position.

$$\begin{aligned} &\varphi_1 = \frac{1}{2} \cdot \left(\varphi_0 \cdot \cos(\omega_+ t) + \varphi_0 \cdot \cos(\omega_- t)\right) \\ &\varphi_2 = \frac{1}{2} \cdot \left(\varphi_0 \cdot \cos(\omega_+ t) - \varphi_0 \cdot \cos(\omega_- t)\right) \end{aligned} \tag{6}$$

After rearranging the equations they take the form

$$\begin{aligned} &\varphi_1 = \varphi_0 \cdot \cos(\omega_\Delta t) \cdot \cos(\omega t) \\ &\varphi_2 = \varphi_0 \cdot \sin(\omega_\Delta t) \cdot \cos(\omega t) \end{aligned} \tag{7}$$

with

$$\begin{aligned} &\omega_\Delta = \frac{\omega_- - \omega_+}{2} \\ &\omega = \frac{\omega_+ + \omega_-}{2} \end{aligned} \tag{8}$$

This corresponds to an oscillation of both pendulums at identical angular frequency ω, where the amplitudes are modulated at an angular frequency ω_Δ. This kind of modulation results in beats. In the situation described, the amplitude of the beats arrives at a maximum since the overall amplitude falls to a minimum at zero.

LIST OF APPARATUS

2 Pendulum Rods with Angle Sensor @230 V 1000763 (U8404275-230)
or
2 Pendulum Rods with Angle Sensor @115 V 1000762 (U8404275-115)
1 Helical Spring 3.0 N/m 1002945 (U15027)
2 Table Clamps 1002832 (U1326)
2 Stainless Steel Rods 1000 mm 1002936 (U15004)
1 Stainless Steel Rod 470 mm 1002934 (U15002)
4 Universal Clamps 1002830 (U13255)
2 HF Patch Cords, BNC/4 mm Plug 1002748 (U11257)
1 3B NETlog™ @230 V 1000540 (U11300-230)
or
1 3B NETlog™ @115 V 1000539 (U11300-115)
1 3B NETlab™ 1000544 (U11310)
1 PC with Windows, Internet Explorer, USB port

SET-UP

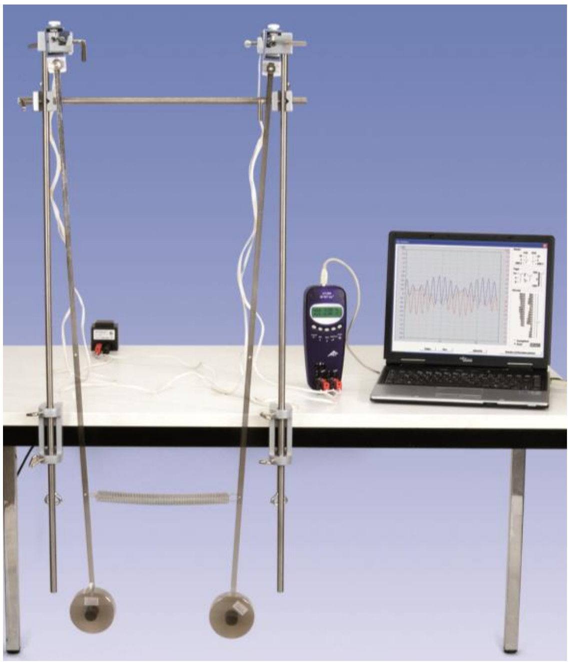


Fig. 2 Set-up for recording and evaluating the oscillation of two identical pendulums coupled together by a spring

The set-up is illustrated in Fig. 2.

- Clamp two stand rods of 1000 mm length to a bench so that they are about 15 cm apart.
- Attach a short stand rod between them as a horizontal cross member to lend the set-up more stability.
- Attach the angle sensors to the top of the vertical rods using universal clamps.

- Attach bobs to the end of the pendulum rods.
- Suspend the pendulum rods from the angle sensors (there are grooves in the angle sensors to accommodate the hinge pins of the pendulum rods)
- Attach the spring via the holes in the pendulum rods. These are about 40 cm from the fulcrum of the pendulum.
- Attach the angle sensor by means of cables to a transformer. Make sure to use the cables labelled 12 V.
- Connect the 3B NET*log*™ equipment to a computer.
- Connect the two angle sensors to the voltage inputs of the 3B NET*log*™ device making sure to keep the correct polarity (red: "+" pole, black: "-" pole).

EXPERIMENT PROCEDURE

- Turn on the 3B NET*log*™ equipment and run 3B NET*lab*™ on the computer.
- Select "Measuring lab" and create a new data entry.
- Select analog inputs A and B and set both to a range of 2 V in DC mode (Vdc).
- Set the following parameters for the measurement: Rate: 50 Hz, No. of measurements: 600, Mode: Standard

1. Record an in-phase oscillation

- Deflect both pendulums to the same (small) angle and release them simultaneously.
- Start the measurement in 3B NET*lab*™.
- After the measurement is complete, select "Reset" and save the results under some meaningful name.

2. Record an out-of-phase oscillation

- Deflect both pendulums to the same (small) angle but in opposite directions and release them simultaneously
- Start measuring in 3B NET*lab*™ again.
- After the measurement is complete, select "Reset" and save the results under a new name.

3. Record the oscillation of coupled pendulums with maximum beat amplitude

- Select "Change settings" and increase the number of measurements to 1200.
- Deflect one pendulum rod keeping the other in its rest position then release both together.
- Start measuring in 3B NET*lab*™ again.
- After the measurement is complete, select "Reset" and save the results under a new name.

SAMPLE MEASUREMENTS

1. In-phase coupled oscillation

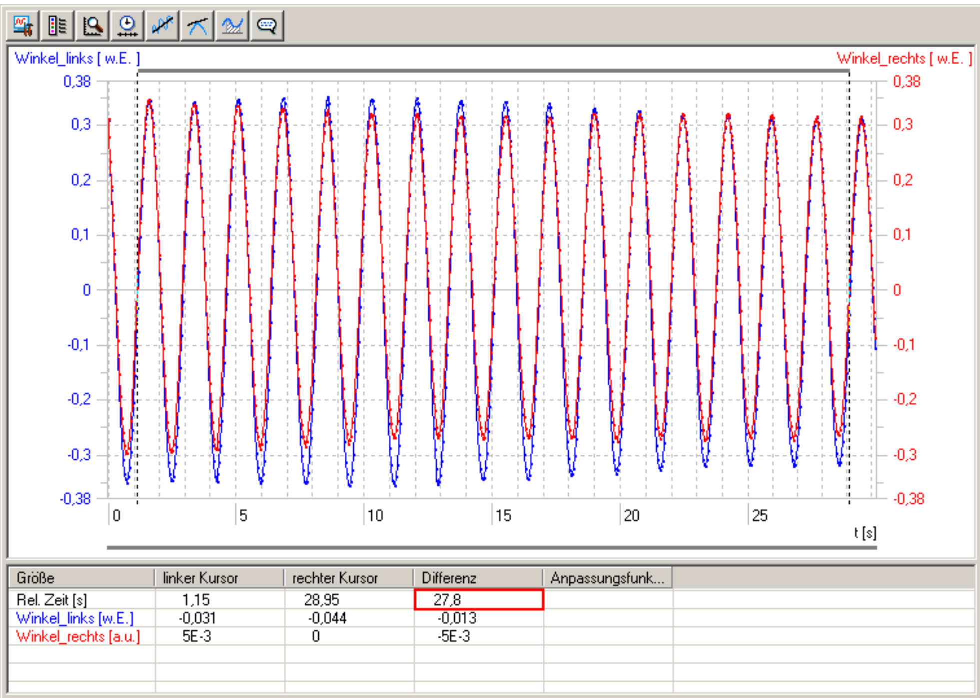


Fig. 3 Angle-time diagram for an in-phase oscillation of coupled pendulums (blue: left-hand pendulum, red: right-hand pendulum). The angle scale has not been calibrated.

2. Out-of-phase coupled oscillation

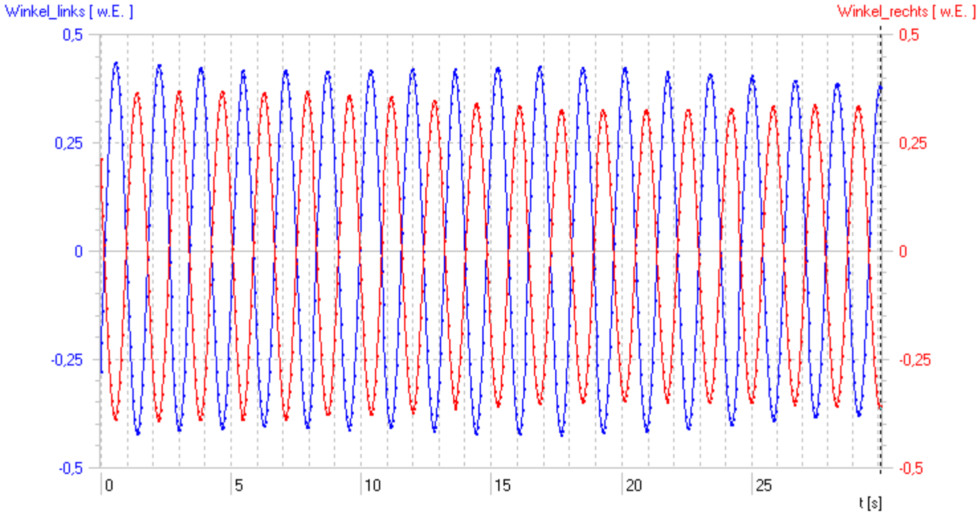


Fig. 4 Angle-time diagram for an out-of-phase oscillation of coupled pendulums (blue: left-hand pendulum, red: right-hand pendulum). The angle scale has not been calibrated.

3. Oscillation of coupled pendulums with maximum beat amplitude

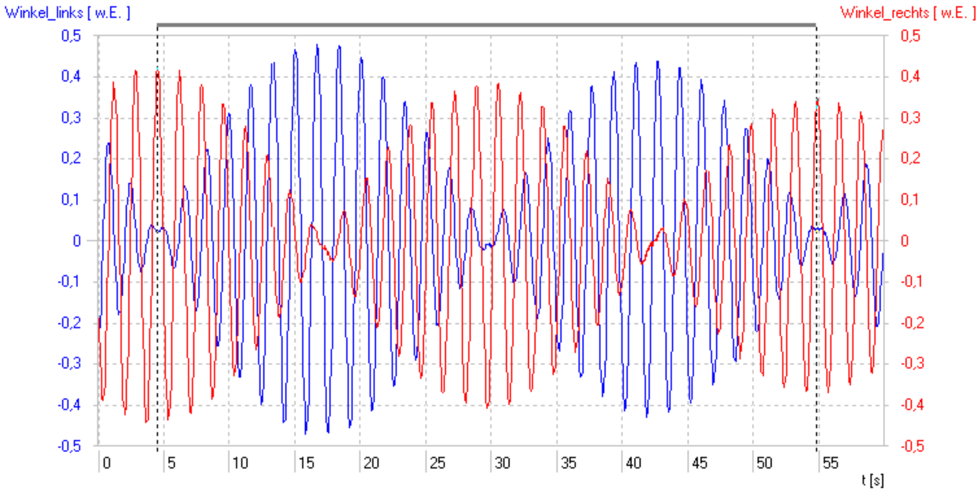


Fig. 5 Angle-time diagram for an oscillation of coupled pendulums with maximum beat amplitude (blue: left-hand pendulum, red: right-hand pendulum). The angle scale has not been calibrated.

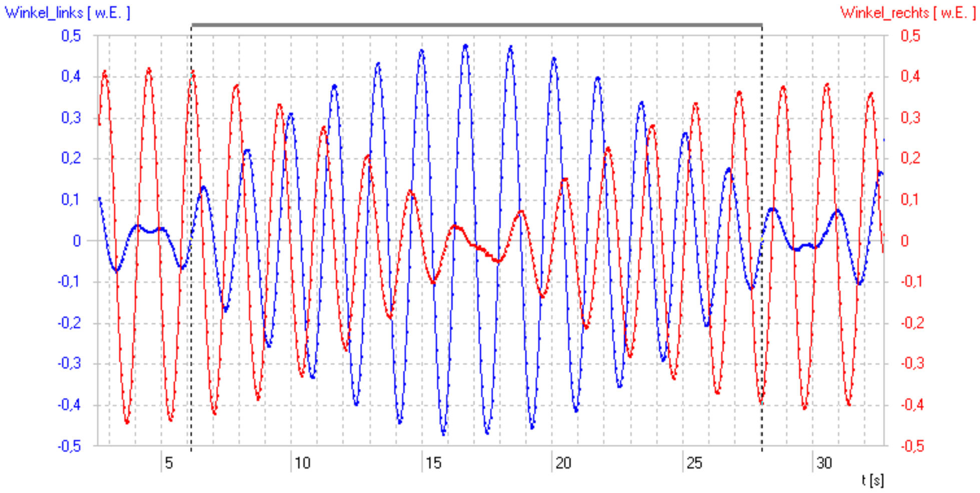


Fig. 6: Magnified view of one beat period in the oscillation of coupled pendulums with maximum beat amplitude (blue: left-hand pendulum, red: right-hand pendulum). The angle scale has not been calibrated.

EVALUATION

1. Determine the period of oscillation for coupled pendulums oscillating in phase

- Open the data entry for the in-phase oscillation.
- Set up the display to include as many complete oscillations as possible between cursors. The cursors should be set precisely at points where the oscillation crosses the axis heading upwards so that a whole number of periods is included (cf. Fig. 3).
- Read off the time between the cursors from the table under the display (Fig. 3, red box).

The period of the oscillation is the time between the cursors divided by the number of complete oscillations included in that time

$$T_+ = \frac{27.8\text{ s}}{16} = 1.737\,s$$

2. Determine the period of oscillation for coupled pendulums oscillating out of phase

- Open the data entry for the out-of-phase oscillation and proceed exactly as before.

The period of the oscillation is the time between the cursors divided by the number of complete oscillations included in that time

$$T_{,} = 1.629\,s$$

3. Determine the period of oscillation for coupled pendulums oscillating with a maximum beat amplitude

- Open the data entry for the oscillation with the maximum beat amplitude.
- Set up the cursors so that they include one or more complete periods of the beat oscillation (cf. Fig. 5) and read off the time from the table underneath.

The period of the maximum beat amplitude is the time between the cursors divided by the number of periods of the beat oscillation included in that time

$$T_\Delta = 25\,s$$

- Change the scale of the time axis so that one period of the beats is displayed in a magnified view.
- Set the cursors so that they include as many oscillations of one of the pendulums as possible within the space of one beat (time between successive points where the pendulums stop still at the rest position - cf. Fig. 6) and read off the time between the cursors from the table at the bottom.

The period of the oscillation is the time between the cursors divided by the number of complete oscillations included in that time

$$T = 1.685\,s$$

4. Compare the measurements for the two periods with the values as calculated from the intrinsic periods

For a coupled oscillation with a period *T*, where the beat amplitude is at its maximum, the expression below follows from equation (8):

$$T = 2\cdot\frac{T_+\cdot T_{,}}{T_+ + T_{,}} = 1.681\,s \qquad (9)$$

Compare this with the measured value of T = 1.685 s.

The beat period T_Δ can also be calculated in a similar way. It should be noted, however, that this is usually defined to be the time between successive points where the pendulums stop still at the rest position. This actually represents only half the period of the underlying cosine or sine modulation term in equation (7).

$$T_\Delta = \frac{T_+\cdot T_{,}}{T_+ - T_{,}} = 26\,s \qquad (10)$$

Compare this answer with the measured value of $T_\Delta = 25\,s$.

The difference of about one second between the calculated and measured values may seem to be quite large at first glance but it is due to the calculation being highly sensitive to differences between the intrinsic oscillation periods. If the intrinsic periods of the two pendulums differ by as little as 4 milliseconds, which roughly corresponds to the maximum measurement accuracy achievable for the intrinsic oscillation periods in this experiment, it leads to a difference of a whole second in the beat period.

5. Determine the spring constant of the spring coupling the two pendulums

The spring constant *D* of the spring coupling the pendulums is related to the coupling constant *k* as follows:

$$D = k\cdot\frac{L}{d^2}\cdot m \qquad (11)$$

(*d*: distance between the point at which the spring is connected to the pendulum and the fulcrum of the pendulum)

If the coupling is weak (*k* << *g*) the spring constant has little influence on the period of the out-of-phase oscillation but has a major influence on the beat period. Thus to calculate the spring constant, we relate it to the beat period by substituting equation (4) into (8) and rearranging to give *k*.

$$k = 2\cdot L\cdot\left(\omega_\Delta^2 - \omega_\Delta\cdot\omega_+\right) \qquad (12)$$

Now the angular frequencies are replaced by the periods and substituted into equation (11) to give:

$$D = \frac{L}{d^2}\cdot m\cdot\frac{g}{2}\cdot\left(2\cdot\frac{T_+}{T_\Delta} + \frac{T_+{}^2}{T_\Delta{}^2}\right) = 3.5\,\frac{\text{N}}{\text{m}} \qquad (13)$$