

Moment of Inertia

DETERMINE THE MOMENT OF INERTIA FOR VARIOUS TEST BODIES

- Determine the torsional coefficient D_r between for the springs used to couple the objects.
- Determine the moment of inertia J for a dumbbell bar without any added weights
- Determine the moment of inertia J as a function of distance r of a weight from its axis of rotation.
- Determine the moment of inertia J for a circular wooden disc, a wooden sphere and both solid and hollow cylinders
- Verify the parallel-axis/Huygens-Steiner theorem

UE1040205

07/15 UD



Fig. 1: Experiment set-up.

GENERAL PRINCIPLES

The inertia of a rigid body with respect to a change in its rotational motion around a fixed axis is given by its moment of inertia J . This is dependent on the distribution of mass in the body relative to the axis of rotation and increases at greater distance from the axis of rotation itself.

In general, moment of inertia is defined by means of a volume integral:

$$(1) \quad J = \int_V r_s^2 \rho(\mathbf{z}) \cdot dV$$

r_s : component of r perpendicular to the axis of rotation

$\rho(r)$: Distribution of mass in the body

Using as an example a dumbbell set, which has two weights of mass m symmetrically arranged at a distance r from the axis of rotation, then the moment of inertia is as follows:

$$(2) \quad J = J_0 + J_m = J_0 + 2 \cdot m \cdot r^2$$

J_0 : Moment of inertia of dumbbell bar without weights

J_m : Moment of inertia of weights

Now we can attach various test bodies to a twisting axis so that they can oscillate. If the period of oscillation is T , then the following is true:

$$(3) \quad T = 2\pi \cdot \sqrt{\frac{J}{D_r}}$$

D_r : Torsional coefficient of coil springs

The means that the period of oscillation T will be greater when the moment of inertia J is larger.

From equation (3) it is possible to obtain a formula for determining the moment of inertia:

$$(4) \quad J = D_r \cdot \frac{T^2}{4\pi^2}$$

The torsional coefficient of the coil springs can be determined with the help of a spring dynamometer:

$$(5) \quad D_r = \frac{F \cdot r}{\alpha}$$

α : Deflection from equilibrium state

LIST OF EQUIPMENT

1 Torsion Axle	U20050	1008662
1 Photo Gate	U11365	1000563
1 Digital Counter	U8533341	1001032/3
1 Barrel Foot, 1000 g	U13265	1002834
1 Tripod Stand 185 mm	U13271	1002836
1 Precision Dynamometer 1 N	U20032	1003104
1 Set of Test Bodies for Torsion Axle	U20051	1008663

SET UP AND PROCEDURE

- Set up the experiment as shown in Fig. 1. Align the rotational axis to be horizontal to the stand base with the help of the spirit level and the levelling screws.
- Connect the photo gate to channel A of the digital counter. Set the operating mode selector switch on the digital counter to the symbol for measuring the periods of a pendulum.

Notes:

- When deflecting the bar, do so in such a way that the coupling springs are forced together and do not bend outwards.
- At the start of the oscillation, it is recommended that a deflection angle of 180° be used (max. 360°).

Determining torsional coefficient D_r for coupling springs

- Suspend the spring dynamometer at distances $r = 5, 10, 15, 20, 25$ and 30 cm from the centre of the rotational axis in sequence on the dumbbell bar and deflect them by an angle $\alpha = 180^\circ = \pi$. Make sure that the dynamometer hangs down at right angles to the dumbbell bar.
- Read off the force values needed to deflect the bar at the various distances from the dynamometer. Enter all the results into Table 1.

Determining the moment of inertia J_0 for the bar without weights

- Deflect the dumbbell bar without any weights attached by an angle of 180° and use the digital counter to measure the period of oscillation T_0 .

Determination of moment of inertia J as a function of the distance of weights r from the axis of rotation

- Attach the two weights to the bar symmetrically at distances $r = 5, 10, 15, 20, 25$ and 30 cm to the left and right of the axis of rotation.
- Do not touch the screws which press the ratchet balls against the dumbbell bar. They are adjusted in such a way that the weights can be moved but can also be secured against centrifugal force.
- Deflect the bar by 180° and use the digital counter to measure the periods of oscillation T for each distance. Enter the results into Table 2.

Determine the moments of inertia J for a wooden disc, a wooden sphere, a solid cylinder and a hollow cylinder

- Attach the sample bodies to the torsion axle one after the other. Use the pan for the solid cylinder and the hollow cylinder.
- In order to measure the length of a period of oscillation, use a suitable method to attach a paper flag to each of the samples in order to break the photo gate beam.
- First deflect the wooden disc by 180° then fit the wooden sphere and do the same. In each case, measure the period of oscillation and enter the results into Table 3. Use the white markings on the samples to help you with the orientation for the deflection.
- Deflect the pan by 180° and measure the period of oscillation. Enter the value into Table 3.
- Put the solid cylinder on the pan, deflect it by 180° and measure the period of oscillation. Do the same for the hollow cylinder and enter all the results into Table 3. Use the white markings on the samples to help you with the orientation for the deflection.

Verification of Huygen-Steiner theorem

- Fasten the bolt through the boreholes at distances $a = 0, 2, 4, 6, 8, 10, 12$ and 14 cm from the centre of the disc
- For each of the various positions of the bolt, mount the disc onto the axis, deflect it by 180° in each case and measure the period of oscillation. Use a suitable method to attach a paper flag to the disc. Enter the values into Table 4.

SAMPLE MEASUREMENT

Determining torsional coefficient D_r for coupling springs

Tab. 1: Measurements of force F at a distance r from the centre of the axis of rotation when the dumbbell bar is deflected and allowed to oscillate from a static position $\alpha = 180^\circ = \pi$ from its rest position.

r / m	F / N
0.05	1.72
0.10	0.86
0.15	0.58
0.20	0.46
0.25	0.32
0.30	0.26

Determination of moment of inertia J_0 for dumbbell bar without weights

Period of oscillation T_0 : 2460 ms

Determination of moment of inertia J as a function of the distance of weights r from the axis of rotation

Tab. 2: Period of oscillation T for a dumbbell bar with weights attached at a distance r from the axis of rotation

r / m	T / ms
0.05	2825
0.10	3663
0.15	4740
0.20	5926
0.25	7170
0.30	8440

Determination of moment of inertia J for a wooden disc, a wooden sphere, a solid cylinder and a hollow cylinder

Tab. 3: Period of oscillation T for various sample bodies

Sample bodies	T / ms
Disc	1800
Sphere	1880
Pan	512
Solid cylinder + pan	917
Hollow cylinder + pan	1171

Verification of Huygens-Steiner theorem

Tab. 4: Period of oscillation T for oscillation of a wooden disc around points at various distances a from its centre of gravity.

a / cm	T / ms
0	2922
2	2960
4	3121
6	3327
8	3622
10	3948
12	4359
14	4748

EVALUATION

Determining torsional coefficient D_r for coupling springs

From equation (5), the following applies:

$$(6) \quad F = \alpha \cdot D_r \cdot \frac{1}{r} = C \cdot \frac{1}{r} \quad \text{where } C = \alpha \cdot D_r$$

- Plot the force measurements F from Table 1 against the inverse of the distances $1/r$ and draw a straight line through the measurement points.

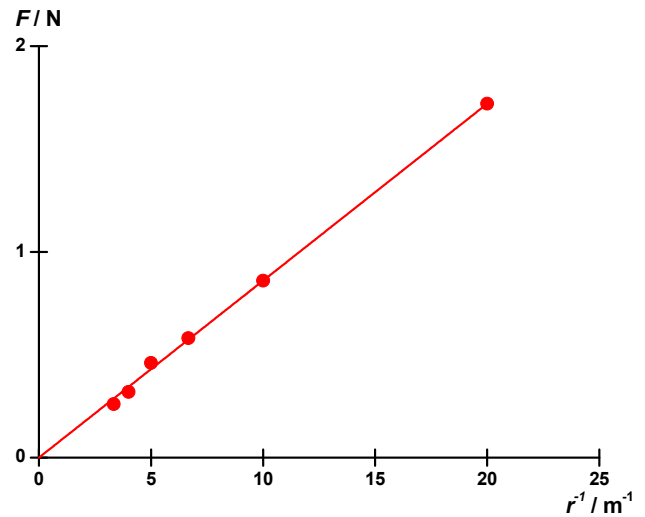


Fig. 2: Force F as a function of the inverse of the distance of the weights from the centre $1/r$.

- Determine the torsional coefficient D_r from the gradient C as indicated in equation (6):

$$(7) \quad C = \alpha \cdot D_r \Leftrightarrow D_r = \frac{C}{\alpha} = \frac{0.0860 \text{ Nm}}{\pi} = 0.0274 \text{ Nm}$$

Determination of moment of inertia J_0 for dumbbell bar without weights

From equation (4), the moment of inertia for the dumbbell bar without weights is given by:

$$(8) \quad J_0 = 0.0274 \text{ Nm} \cdot \frac{(2.460 \text{ s})^2}{4\pi^2} = 4.20 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2.$$

Determination of moment of inertia J as a function of the distance of weights r from the axis of rotation

- Use equation (4) to calculate the moments of inertia J of the dumbbell bar with weights from the values in Table 2 and enter them into Table 5.
- Determine the moments of inertia of the weights themselves J_m from the following

$$(9) \quad J_m = J - J_0$$

Enter the results into Table 5

Tab. 5: Period T , moment of inertia J for dumbbell bar with weights plus moment of inertia J_m for weights at various distances r from the axis of rotation.

r / m	T / s	$J / 10^{-3} \text{ kg} \cdot \text{m}^2$	$J_m / 10^{-3} \text{ kg} \cdot \text{m}^2$
0.05	2.825	5.54	1.34
0.10	3.663	9.31	5.11
0.15	4.740	15.6	11.4
0.20	5.926	24.4	20.2
0.25	7.170	35.7	31.5
0.30	8.440	49.4	45.2

According to (2):

$$(10) \quad J_m = 2 \cdot m \cdot r^2$$

- Plot the moments of inertia J_m from Table 5 against the square of the distance r^2 and confirm the linear relationship expected from equation (10) (Fig. 3).

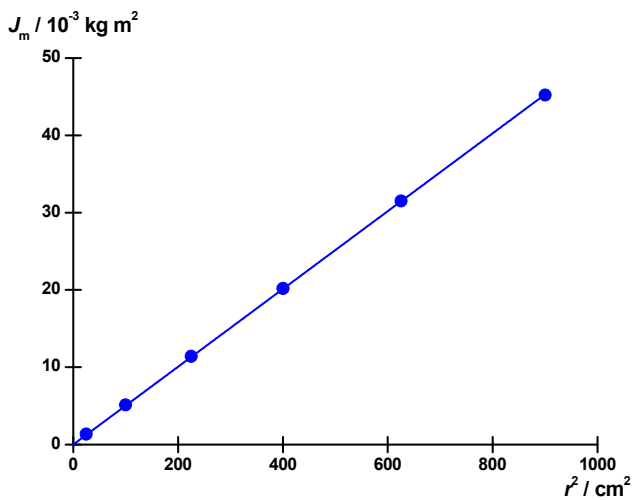


Fig. 3: Moment of inertia of weights J_m as a function of the distance of the weights from the axis r .

Determination of moment of inertia J for a wooden disc, a wooden sphere, a solid cylinder and a hollow cylinder

- Using equation (4), calculate the moments of inertia J for the various sample bodies and the measurements entered into Table 3 and enter the results into Table 6.
- In order to determine the moments of inertia of the solid cylinder and the hollow cylinder J_V and J_H , in each case subtract the moment of inertia of the pan J_T from the values of the moment of inertia for the solid cylinder + pan or the hollow cylinder + pan J_{VT} and J_{HT} :

$$(11) \quad \begin{aligned} J_V &= J_{VT} - J_T \\ J_H &= J_{HT} - J_T \end{aligned}$$

- Calculate the theoretical values for the moment of inertia J_{th} with the help of the data in the appendix. Enter them into Table 6 and compare them with the values determined by measurement.

Tab. 6: Moments of inertia J for various sample bodies.

Sample bodies	T / s	$J / 10^{-3} \text{ kg} \cdot \text{m}^2$	$J_{th} / 10^{-3} \text{ kg} \cdot \text{m}^2$
Disc	1.800	2.25	$1/2 \cdot m \cdot r^2 = 2.57$
Sphere	1.880	2.45	$2/5 \cdot m \cdot r^2 = 2.54$
Pan	0.512	0.18	–
Solid cylinder + pan	0.917	0.58	–
Solid cylinder	–	0.40	$1/2 \cdot m \cdot r^2 = 0.43$
Hollow cylinder + pan	1.171	0.95	–
Hollow cylinder	–	0.77	$m \cdot r^2 = 0.86$

The values determined by measurement are well in agreement with those calculated in accordance with the theory.

Verification of Huygens-Steiner theorem

- Determine the moments of inertia J_a for various distances a from the measurements in Table 4 using equation (4) and enter these values into Table 7.

Tab. 7: Moment of inertia J_a for a disc oscillating about various axes at distance a from its centre of gravity.

a / cm	T / s	$J_a / 10^{-3} \text{ kg} \cdot \text{m}^2$
0	2.922	5.93
2	2.960	6.08
4	3.121	6.76
6	3.327	7.68
8	3.622	9.11
10	3.948	10.8
12	4.359	13.2
14	4.748	15.6

- Gemäß dem Steiner'schen Satz gilt:
(12) $J_a = J_0 + m \cdot a^2$ where $J_0 = J_a (a = 0)$
- Plot $J_a - J_0$ against a^2 , and confirm the linear relationship indicated in equation (12), thereby verifying the Huygens-Steiner theorem (Fig. 4).

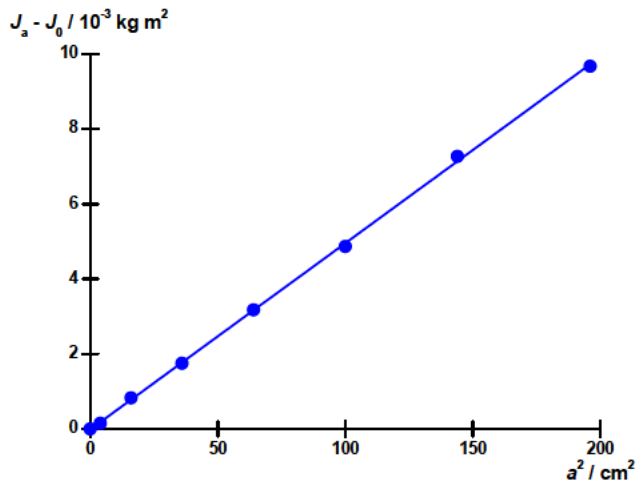


Fig. 4: Difference in moments of inertia $J_a - J_0$ of a disc as a function of the distance a of the axis of rotation from its centre of gravity.

APPENDIX: TECHNICAL DATA

Dumbbell bar

Length:	620 mm
Weight:	approx. 135 g
Weights:	260 g each

Disc

Diameter:	320 mm
Weight:	approx 495 g
Boreholes:	8
Borehole spacing:	20 mm

Wooden sphere

Diameter:	146 mm
Weight:	approx. 1190 g

Wooden disc

Diameter:	220 mm
Height:	15 mm
Weight:	approx. 425 g

Mounting plate

Diameter:	100 mm
Weight:	approx. 122 g

Solid cylinder (wood)

Diameter:	90 mm
Height:	90 mm
Masse	approx. 425 g

Hollow cylinder (metal)

External diameter:	90 mm
Height:	90 mm
Weight:	approx. 425 g