

EXPERIMENT PROCEDURE

- Measuring the width of the trajectory as a function of the throwing angle and the initial velocity.
- Calculating the initial velocity from the maximum width of the trajectory.
- Point-by-point plotting of the “parabolic” trajectory as a function of the throwing angle and the initial velocity.
- Verification of the principle of superposition.

OBJECTIVE

Plotting the “parabolic” trajectories point by point

SUMMARY

The motion of a ball that is thrown upward at an angle to the horizontal in the earth’s gravitational field follows a parabolic curve whose height and width depend on the throwing angle and the initial velocity. The curve is measured point by point using a height scale with two pointers.

REQUIRED APPARATUS

Quantity	Description	Item Number
1	Projectile Launcher	1002654
1	Clamp for Projectile Launcher	1002655
1	Vertical Ruler, 1 m	1000743
1	Set of Riders for Rulers	1006494
1	Barrel Foot, 900 g	1002834
1	Pocket Measuring Tape, 2 m	1002603

BASIC PRINCIPLES

According to the principle of superposition, the motion of a ball that is thrown upward at an angle to the horizontal in the earth’s gravitational field is the combination of a motion at a constant speed in the direction of throwing and a gravitational falling motion. This results in a parabolic flight curve, whose height and width depend on the throwing angle  $\alpha$  and the initial velocity  $v_0$ .

To calculate the theoretical flight curve, for simplicity we take the center of the spherical ball as the origin of the coordinate system, and we neglect the frictional drag of the air on the ball. Thus the ball retains its initial velocity in the horizontal direction

$$(1) \quad v_x(0) = v_0 \cdot \cos \alpha$$

and therefore at time  $t$  the horizontal distance travelled is

$$(2) \quad x(t) = v_0 \cdot \cos \alpha \cdot t$$

In the vertical direction, under the influence of the gravitational field, the ball is subjected to gravitational acceleration  $g$ . Therefore, at time  $t$  its vertical velocity is

$$(3) \quad v_y(t) = v_0 \cdot \sin \alpha - g \cdot t$$

and the vertical distance travelled is

$$(4) \quad y(t) = v_0 \cdot \sin \alpha \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

The flight curve of the ball has the form of a parabola, as it conforms to the equation

$$(5) \quad y(x) = \tan \alpha \cdot x - \frac{1}{2} \cdot \frac{g}{(v_0 \cdot \cos \alpha)^2} \cdot x^2$$

At time  $t_1$  given by

$$(6) \quad t_1 = \frac{v_0 \cdot \sin \alpha}{g}$$

the ball reaches the highest point of the parabola, and at time  $t_2$  given by

$$(7) \quad t_2 = 2 \cdot \frac{v_0 \cdot \sin \alpha}{g}$$

it is again at the initial height 0. Thus, the height of the parabola is

$$(8) \quad h = y(t_1) = \frac{v_0^2}{2 \cdot g} \cdot \sin^2 \alpha$$

and the width is

$$(9) \quad s = x(t_2) = 2 \cdot \frac{v_0^2}{g} \cdot \sin \alpha \cdot \cos \alpha$$

In the experiment, the flight curves of a ball are measured point by point as a function of the throwing angle and the initial velocity, using a height scale with two pointers.

EVALUATION

The maximum width of all the flight curves,  $s_{\max}$ , is reached when the throwing angle  $\alpha$  is  $45^\circ$ . From this maximum width, it is possible to calculate the initial velocity. By using Equation 9, we get

$$v_0 = \sqrt{g \cdot s_{\max}}$$

An exact analysis of the experimental data shows that the frictional drag of the air on the ball must be taken into account, and that the flight curves actually depart slightly from the parabolic shape.

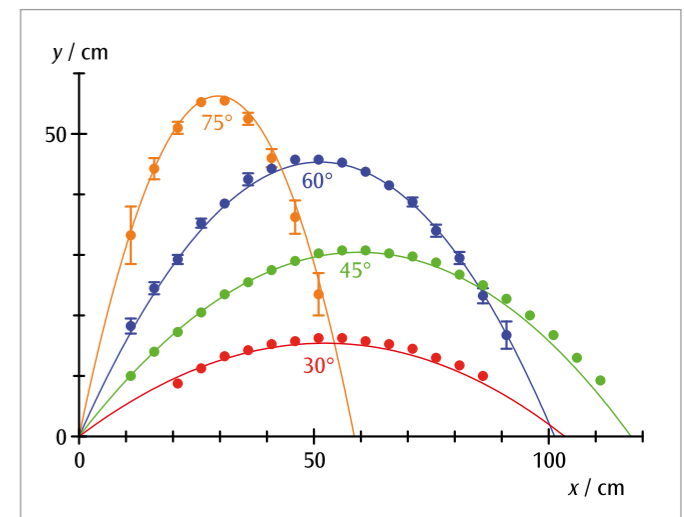


Fig. 1: Flight curves for the smallest initial velocity and different throwing angles, measured experimentally, and calculated theoretically with air friction taken into account