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Ball Pendulum 1017332

Instruction manual

10/15 ALF

1. Description

A rolling ball bearing moves inside a concave acrylic body of spherical curvature. One special case of the ball’s movement is a linear oscillation around its equilibrium position in the nature of a mathematical pendulum. The radius of curvature of the acrylic body then corresponds to the length of the pendulum. One other special case is a circular motion of the ball around the vertical where it acts like a conical pendulum.

Three steel balls are included with the set.

Mathematically speaking, the way the location of the oscillating ball changes over time is described by the space vector in spherical coordinates:

(1) $\vec{r}(t) = (R, \theta(t), \phi(t))$

R: Radius of curvature = Length of pendulum
θ: Polar angle, deflection from equilibrium
ϕ: Angle of azimuth, rotation around vertical

The equation for the potential energy is then

(2) $E_{\text{pot}} = -m \cdot g \cdot R \cdot \cos\theta$

The kinetic energy equation is

(3) $$\begin{aligned} E_{\text{kin}} &= \frac{1}{2} \cdot m \cdot v^2 \\ &= \frac{1}{2} \cdot m \cdot R^2 \left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta \right) \end{aligned}.$$

A point over the term symbolises that it refers to the time differential of the variable. Two related differential equations, containing the variables polar angle θ and azimuth angle ϕ, are derived from those for the potential and kinetic energies. Solutions to these equations include those highlighted below:

1) $\theta = 0$
   The ball is stationary at its equilibrium position in the centre of the acrylic body.

2) $\dot{\phi} = 0$

The ball oscillates around the equilibrium position in a motion equivalent to a mathematical pendulum. The period of oscillation is as follows:

(4) $$T = 2 \cdot \pi \cdot \sqrt{\frac{7}{5} \cdot \frac{R}{g}}.$$

g: Acceleration due to gravity

3) $\dot{\phi} = \sqrt{\frac{g}{R \cdot \cos\theta}}$

The ball rotates around the vertical in a motion equivalent to a conical pendulum.

2. Technical data

Radius of curvature:	200 mm
Diameter:	140 mm
Diameter of ball:	16 mm

3. Operation

- Cause the ball to oscillate in the ways specified in the description as special cases 2) and 3).
- For the mathematical pendulum of special case 2), measure the period of oscillation T with the help of a stop watch and verify that equation (4) is true.

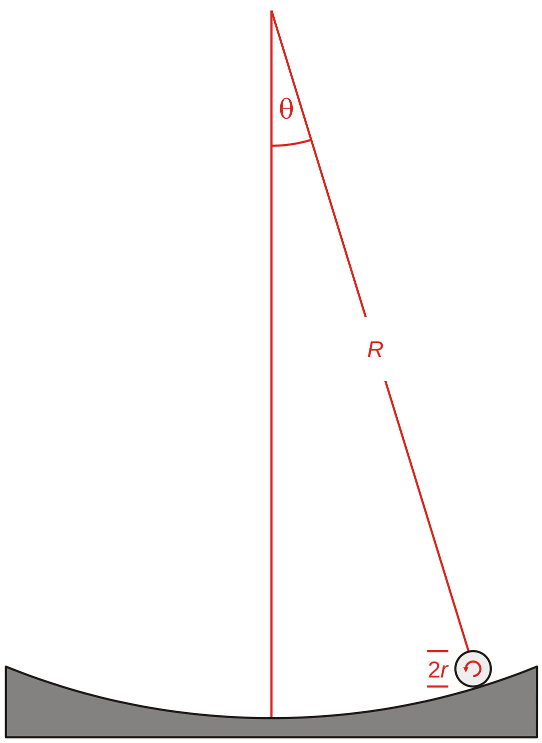

Fig. 1 Schematic of ball pendulum