Moment of inertia

DETERMINE THE MOMENT OF INERTIA OF A HORIZONTAL ROD WITH ADDITIONAL WEIGHTS ATTACHED.

- Determine the torsional coefficient D_r of the coupled spring.
- Determine the moment of inertia J as a function of the distance r of the added weights from the axis of rotation.
- Determine the moment of inertia J as a function of the value m of the added weights.

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BASIC PRINCIPLES

The inertia of a rigid body that acts against a change of its rotational motion about a fixed axis is described by the moment of inertia *J*. It depends on the distribution of weight in relation to the axis of rotation. The greater the distance of a weight from the axis of rotation the greater also is the moment of inertia it causes.

In the experiment, this is investigated using the example of a rotating disc carrying a horizontal rod, to which two additional weights of mass m are attached symmetrically at a distance r from the axis of rotation. For this system the moment of inertia is:

$$J = J_0 + 2 \cdot m \cdot r^2 \qquad (1)$$

J_0: moment of inertia without the additional weights

If the rotating disc is coupled elastically by a coil spring to a rigid stand, the moment of inertia can be determined from the period of torsional oscillation of the disc about its rest position. The relationship is as follows:

$$T = 2\pi \cdot \sqrt{\frac{J}{D_r}} \qquad (2)$$

D_r: torsional coefficient of the coil spring

Thus, the greater the moment of inertia J of the disc with the attached horizontal rod, as dependent on the mass m and the distance r, the longer the period of oscillation T.

Fig. 1: Experiment set-up for determining the moment of inertia by the torsional oscillation method

LIST OF APPARATUS

1 Rotating System on Air Bed U8405680
1 Supplementary Kit for Rotating System on Air Bed U8405690

1 Laser Reflection Sensor U8533380

1 Digital Counter U8533341

SET-UP

- Set up the "rotating system on air bed" as described in the instruction sheet, and level it horizontally.
- Place on it the rotating disc with the horizontal rod and screw on the graduated pulley.
- Place the laser reflection sensor on the side-bracket of the start/stop unit and connect it to the "start" input of the digital counter.
- Switch on the air blower and move the start/stop unit so that its pointer touches the edge of the rotating disc and prevents it from turning freely.
- Rotate the disc until the pointer is at the zero (0°) position.
- Position the laser reflex sensor so that the light beam passes through the hole at the 0° position on the rotating disc.
- Mount the right-angle bracket from the supplementary kit for the rotating system on the system's base tube and fix the universal clamp at the free end.
- Mount the 5 N coupling spring in the universal clamp and couple it to the graduated pulley by means of the permanent magnet.
- Set the function selector switch of the digital counter to the T_A / △ position.

EXPERIMENT PROCEDURE

a) Measurements without additional weights

- Push the disc to set it into torsional oscillation and press the START button of the counter.
- Read off the oscillation period, repeating the measurement several times, and enter the average value *T* in the first line of Table 1.

b) Measurements with additional weights

- Hang two additional weights, each of value *m* = 50 g, symmetrically from the horizontal rod at a distance *r* = 30 mm from the axis of rotation.
- Calculate the period of oscillation as the average of several measurements and enter it in Table 1.
- Increase the distance *r* in steps of 20 mm, measure the period of oscillation *T* in each case and enter the results in Table 1.
- Carry out two similar series of measurements with 25 g and 12.5 g as the additional weights and enter the results in Table 1.

SAMPLE MEASUREMENTS

Table 1: Experiment data

m / g	*r* / cm	r^2 / cm²	*T* / s	T^2 / s²	*J* / g m²
	0	0	6.002	36.02	0.89
50	3	9	6.310	39.81	0.98
50	5	25	6.807	46.34	1.14
50	7	49	7.485	56.02	1.38
50	9	81	8.320	69.22	1.70
50	11	121	9.237	85.32	2.10
50	13	169	10.238	104.81	2.58
50	15	225	11.294	127.54	3.14
50	17	289	12.402	153.81	3.78
50	19	361	13.538	183.26	4.51
50	21	441	14.683	215.59	5.30
25	3	9	6.149	37.81	0.93
25	5	25	6.411	41.10	1.01
25	7	49	6.770	45.83	1.13
25	9	81	7.230	52.28	1.29
25	11	121	7.772	60.40	1.48
25	13	169	8.365	69.97	1.72
25	15	225	9.009	81.15	2.00
25	17	289	9.711	94.29	2.32
25	19	361	10.423	108.64	2.67
25	21	441	11.174	124.87	3.07
12.5	3	9	6.074	36.90	0.91
12.5	5	25	6.203	38.48	0.95
12.5	7	49	6.399	40.95	1.01
12.5	9	81	6.653	44.27	1.09
12.5	11	121	6.950	48.30	1.19
12.5	13	169	7.303	53.33	1.31
12.5	15	225	7.673	58.88	1.45
12.5	17	289	8.078	65.25	1.60
12.5	19	361	8.522	72.62	1.79
12.5	21	441	8.995	80.91	1.99

EVALUATION

From (2) the following equation is derived to determine the moment of inertia:

$$J = D_r \cdot \frac{T^2}{4\pi^2}$$

However, D_r is unknown at first. Its value can be calculated by using the following equation:

$$D_r \cdot \frac{T^2 - T_0^2}{4\pi^2} = J - J_0 = 2 \cdot m \cdot r^2$$

This can be rearranged as follows and then with the values shown in red in Table 1 inserted:

$$D_r \cdot = 2 \cdot m \cdot r^2 \cdot \frac{4\pi^2}{T^2 - T_0^2}$$

$$= 2 \cdot 50\ \text{g} \cdot 441\ \text{cm}^2 \cdot \frac{4\pi^2}{215.59\ \text{s}^2 - 36.02\ \text{s}^2} = 970\ \frac{\text{mN} \cdot \text{mm}}{\text{rad}}$$

By substituting this value of D_r in the expression for J given above, the values in the last column of Table 1 can be calculated.

In Figure 2 the values for the moment of inertia calculated as above are plotted against the square of the distance r of the additional weights from the axis of rotation. The straight lines through the data points have the gradients 2 × 50 g, 2 × 25 g and 2 × 12.5 g.

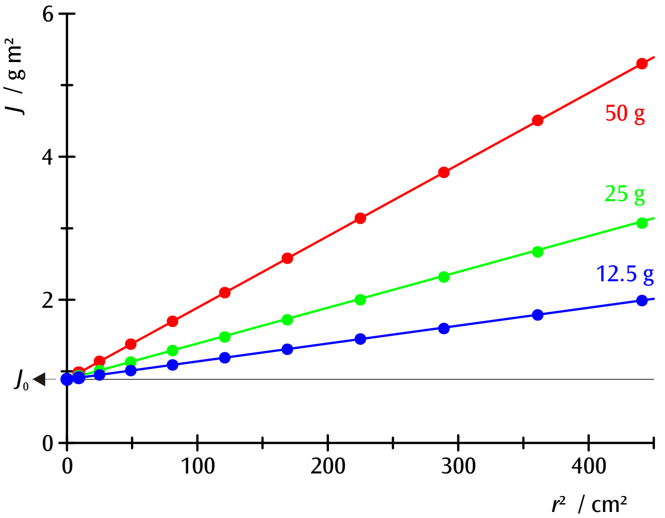

Fig. 2: Moment of inertia J of rotating disc with horizontal rod as a function of the square of the distance r from the axis of rotation for three different additional weights of mass m