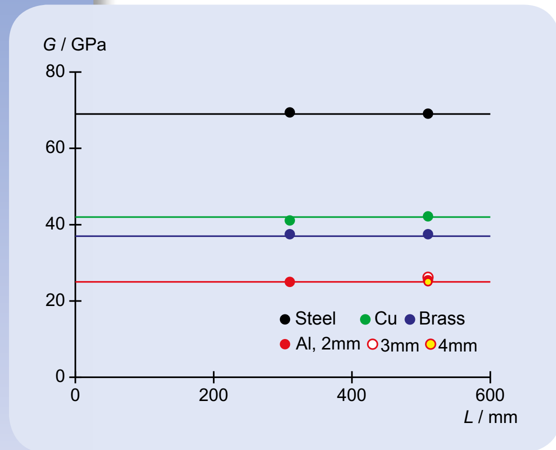

OBJECTIVE

Determination of torsional coefficients and shear modulus

EXPERIMENT PROCEDURE

- **Determine the torsional coefficients of cylindrical rods as a function of their length.**
- **Determine the torsional coefficients of cylindrical rods as a function of their diameter.**
- **Determine the torsional coefficients of cylindrical rods made of various materials and also find their shear modulus.**

You can find technical information about the equipment at 3bscientific.com

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SUMMARY

In order for solid bodies to be deformed, an external force needs to be applied. This acts against the body's own resistance to deformation, which is dependent on the material from which the body is made, as well as its geometry and the direction of the applied force. The deformation is reversible and proportional to the applied force as long as that force is not too great. One example which is often investigated is torsion applied to a uniform cylindrical rod which is fixed at one end. The resistance of the rod to deformation can be numerically analysed and determined by building a set-up which is capable of oscillating involving the rod itself and a pendulum disc and then measuring the period of the oscillation.

REQUIRED APPARATUS

Quantity	Description	Number
1	Torsion Apparatus	1018550
1	Supplementary Set for Torsion Apparatus	1018787
1	Photo Gate	1000563
1	Digital Counter (230 V, 50/60 Hz)	1001033 or
	Digital Counter (115 V, 50/60 Hz)	1001032

BASIC PRINCIPLES

In order for solid bodies to be deformed, an external force needs to be applied. This acts against the body's own resistance to deformation, which is dependent on the material from which the body is made, as well as its geometry and the direction of the applied force. The deformation is elastic, reversible and proportional to the applied force as long as that force is not too great.

One example which is often investigated is torsion applied to a uniform cylindrical rod which is fixed at one end because the resistance of the rod to deformation can be numerically analysed. This involves considering the rod broken down into radial and cylindrical segments of length L. As long as the rod does not bend, then the torsion applied to the rod at the non-fixed end which twists that end of the rod by a small angle ψ causes each of the segments, which are all of radius r, to twist by the following angle:

(1) $$\alpha_r = \frac{r}{L} \cdot \psi$$

(see Fig. 1). The shearing stress would then be:

(2) $$\tau_r = \frac{dF_{r,\varphi}}{dA_{r,\varphi}} = G \cdot \alpha_r$$

G: Shear modulus of the rod's material

The component of the force $dF_{r,\phi}$ acting in tangential direction at the face of the rod:

(3) $$\Delta A_{r,\varphi} = r \cdot d\varphi \cdot dr$$

is given by:

(4) $$dF_{r,\varphi} = G \cdot \frac{r^2}{L} \cdot \psi \cdot d\varphi \cdot dr.$$

It is then easy to calculate the force dF_r required for the torsion to twist the whole of a hollow cylinder of radius r by an angle ψ along with the corresponding torque dM_r:

(5) $$dM_r = r \cdot dF_r = G \cdot 2\pi \cdot \frac{r^3}{L} \cdot \psi \cdot dr$$

Then for a solid rod of radius r_0, the torsion can be found as follows:

(6) $$M = \int_0^{r_0} dM_r = D \cdot \psi \quad \text{where} \quad D = G \cdot \frac{\pi}{2} \cdot \frac{r_0^4}{L}$$

The torque M remains proportional to the angle of twist resulting from the torsion ψ, i.e. the torsional coefficient D is constant, as long as the torque M is not too large. If the torque is too high, then the deformation becomes plastic and irreversible.

In order to determine the torsional coefficient in this experiment, a pendulum disc is coupled to the non-fixed end of the rod. As long as the angle of deflection is not too great, the disc will oscillate about the torsional axis with a period

(7) $$T = 2\pi \cdot \sqrt{\frac{J}{D}},$$

J: Moment of inertia of pendulum disc

As long as the moment of inertia is known, the torsional coefficient can be determined from the period of oscillation. To be more precise, the overall moment of inertia is split into the moment of inertia J_0 for the pendulum disc and the moment of inertia of the two additional weights m, which are situated at a radius R around the torsional axis:

(8) $$J = J_0 + 2 \cdot m \cdot R^2$$

The period of oscillation T for the pendulum disc with the additional weights is then measured along with the period of oscillation T_0 for the pendulum disc without the weights.

EVALUATION

The equation for determining the torsional coefficient is derived from equations (7) and (8) as follows:

$$D = 4\pi^2 \cdot \frac{2 \cdot m \cdot R^2}{T^2 - T_0^2}$$

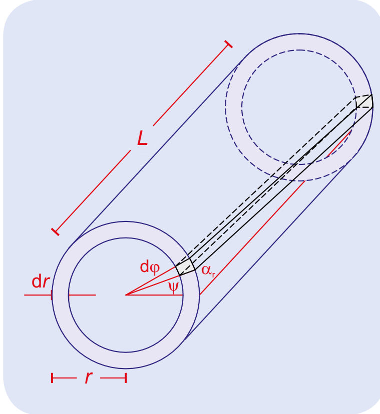

Fig. 1: Schematic for the calculation of the torque dM_r needed to apply torsion on a hollow cylinder of length L, radius r and shell thickness d_r.

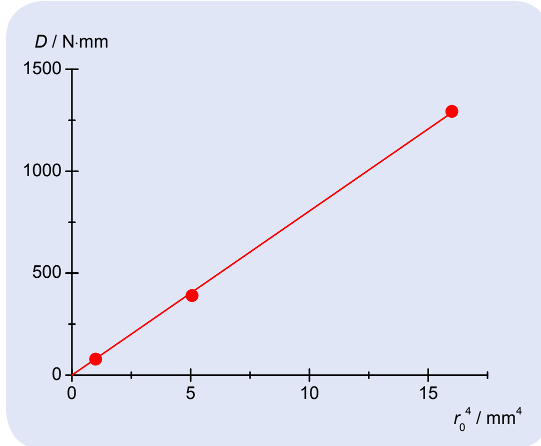

Fig. 2: Torsional coefficient of aluminium rods 500 mm in length as a function of r_0^4.

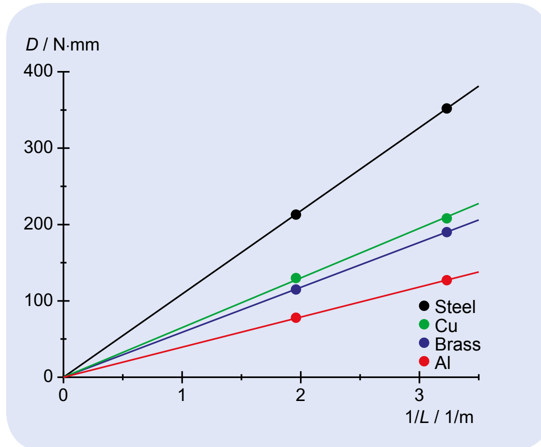

Fig. 3: Torsional coefficient of cylindrical rods as a function of 1/L.