
**OBJECTIVE**

Determine the speeds of sound propagated by longitudinal and transverse waves in solids

**SUMMARY**

In solids, sound is propagated in the form of longitudinal and transverse waves. However, there is a considerable difference in the speed of the two types of sound waves, since longitudinal sound waves are determined by the elastic modulus of the solid, whereas transverse sound waves are dependent on the shear modulus of the solid. By measuring the speed of the two wave types, it is possible to determine the elastic constant of the solid.

**EXPERIMENT PROCEDURE**

- To determine the speed of sound for longitudinal waves in polyacrylic from the propagation time of a 1-MHz ultrasound signal.
- To measure the transmission of longitudinal and transverse sound waves in solids through an inclined, plane-parallel plate.
- To determine the speed of sound for longitudinal and transverse waves from the critical angle of total reflection.
- To determine the elastic modulus  $E$ , the shear modulus  $G$  and Poisson's ratio of a solid  $\mu$  from the two speeds.

**REQUIRED APPARATUS**

Quantity	Description	Number
1	Ultrasonic Echoscope	1002580
2	Ultrasonic Transducers, 1 MHz	1002581
1	Longitudinal and Transversal Wave Equipment Kit	1002584
1	Aluminum Test Block with Protractor Scale	1002585
1	Set of 3 Cylinders	1002588
1	Ultrasonic Coupling Gel	1008575

**BASIC PRINCIPLES**

In gases and liquids, sound is propagated exclusively in the form of longitudinal waves. In the process, the sound pressure oscillates around an equilibrium value and generates oscillating regions of compression and rarefaction. Sound also penetrates solids in the form of transverse waves in which the shear stress oscillates. Transverse waves can propagate through solids because solids possess the necessary shear force required for conducting sound.

Longitudinal and transverse waves possess different speeds which depend on the density  $\rho$  and the elastic constant of the solid. The speed of longitudinal waves, given by

$$(1) \quad c_L = \sqrt{\frac{E}{\rho} \cdot \frac{1-\mu}{(1+\mu)(1-2\mu)}}$$

$E$ : elastic modulus,  $\mu$ : Poisson's ratio

is greater than that of transverse waves

$$(2) \quad c_T = \sqrt{\frac{G}{\rho}}$$

$G$ : shear modulus

**2**

The relation between the elastic modulus  $E$ , shear modulus  $G$  of a solid and Poisson's ratio is given by the following equation:

$$(3) \quad \frac{E}{G} = 2 \cdot (1 + \mu)$$

It is therefore possible to calculate all three magnitudes of elasticity, given that the two sound speeds  $c_L$  and  $c_T$  are known.

In the experiment, first measure the propagation time  $t$  of a 1-MHz ultrasound signal through three polyacrylic cylinders of different lengths  $s$ . Plot the values in an  $s$ - $t$  graph (see Fig. 1). From the inclination of the best-fit line through the measured values, we get the longitudinal sound speed in polyacrylic.

Subsequently, fill a trough with water and place it in the path of the wave. Measure the transit time. The transit time is reduced by placing a thin plane-parallel plate made of polyacrylic or aluminium in the path of the wave. This is due to the fact that sound propagates faster in the plate material than in water. Take accurate readings behind the water trough for the two distinct ultrasound signals caused due to the different propagation times for longitudinal and transversal sound waves in solids (see Fig. 2). If the plate is inclined at an angle  $\alpha$  to the incident wave, then, according to Snell's law, the wave is refracted and the two refracted waves are at angles  $\beta_L$  and  $\beta_T$  (see Fig. 3).

$$(4) \quad \frac{c}{\sin \alpha} = \frac{c_L}{\sin \beta_L} = \frac{c_T}{\sin \beta_T}$$

$c$ : speed of sound in water

As the two sound speeds  $c_L$  and  $c_T$  through the solid are greater than the speed of sound  $c$  in water, we can eventually observe the phenomenon of total reflection – distinctly for longitudinal and transverse waves – in which the transmitted signals fully disappear. The corresponding speeds can be measured from the critical angles  $\alpha_L$  for longitudinal waves and  $\alpha_T$  for transverse waves:

$$(5) \quad c_L = \frac{c}{\sin \alpha_L} \quad \text{and} \quad c_T = \frac{c}{\sin \alpha_T}$$

**EVALUATION**

a) The readings from the first series of propagation time measurements are not on a straight line through the origin on the  $s$ - $t$  graph. This is because the propagation time required by the signal to pass through the adaptation and protective layer of the ultrasonic transducer is also measured systematically.

b) From equations 1 to 3, we get the characteristic equation for Poisson's ratio  $\mu$

$$\mu = \frac{\frac{1}{2} \cdot \left(\frac{c_L}{c_T}\right)^2 - 1}{\left(\frac{c_L}{c_T}\right)^2 - 1}$$

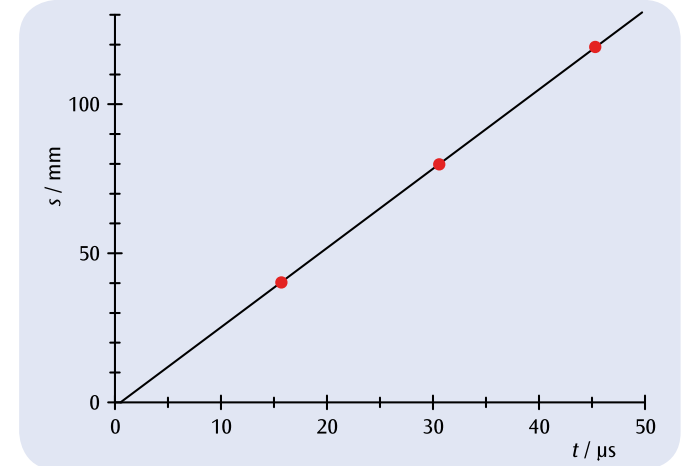


Fig. 1:  $s$ - $t$  graph of an ultrasound signal in polyacrylic

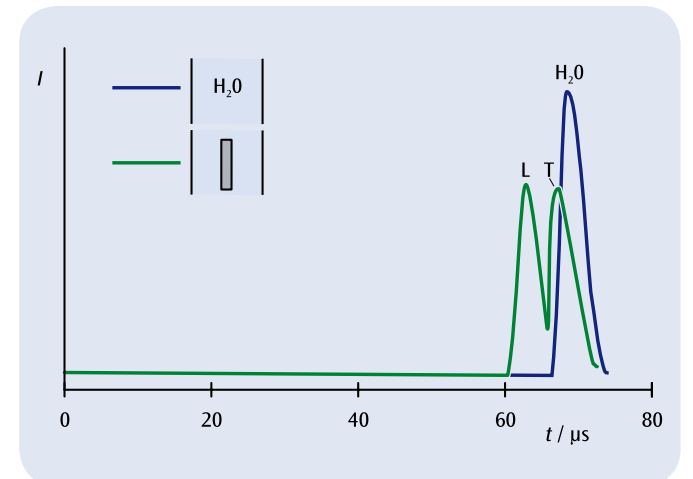


Fig. 2: Ultrasound signal after penetrating a water trough (blue: without plane-parallel plate, green: with plane-parallel plate)

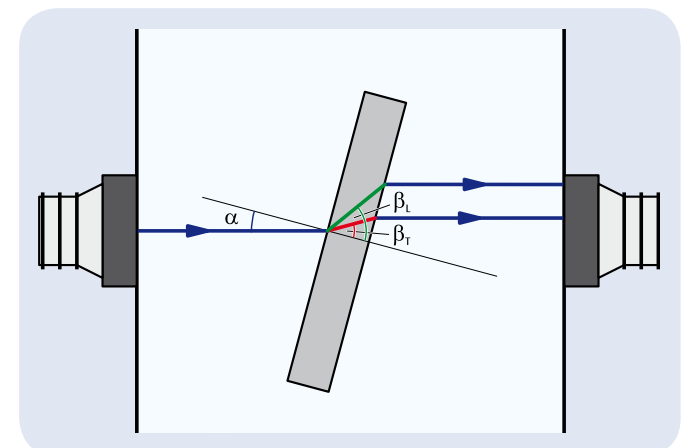


Fig. 3: Experimental set-up for determining the speed of sound for longitudinal and transverse waves from the critical angles of total reflection