COUPLED OSCILLATIONS

EXPERIMENT PROCEDURE

- **Record the oscillations when they are in phase and determine the period T_+.**
- **Record the oscillations when they are out of phase and determine the period T_-.**
- **Record a coupled oscillation and determine the oscillation period T and the beat period T_Δ.**
- **Compare the values obtained for those calculated for the natural periods T_+ and T_-.**

OBJECTIVE

Record and evaluate oscillation of two identical coupled pendulums

SUMMARY

The oscillation of two identical, coupled pendulums is distinguished by the period of oscillation and the beat period. The beat period is the interval between two points in time when one pendulum is swinging at its minimum amplitude. Both values can be calculated from the natural periods of oscillation for the coupled pendulums when the oscillations are in phase and out of phase.

REQUIRED APPARATUS

Quantity	Description	Number
2	Pendulum Rods with Angle Sensor (230 V, 50/60 Hz)	1000763 or
	Pendulum Rods with Angle Sensor (115 V, 50/60 Hz)	1000762
1	Helical Springs 3,0 N/m	1002945
2	Table Clamps	1002832
2	Stainless Steel Rods 1000 mm	1002936
1	Stainless Steel Rod 470 mm	1002934
4	Multiclamps	1002830
2	HF Patch Cord, BNC/4mm plug	1002748
1	3B NET*log*™ (230 V, 50/60 Hz)	1000540 or
	3B NET*log*™ (115 V, 50/60 Hz)	1000539
1	3B NET*lab*™	1000544

BASIC PRINCIPLES

For oscillation of two coupled pendulums, the oscillation energy is transferred from one pendulum to the other and back again. If both pendulums are identical and oscillation is begun so that one pendulum is initially at rest while the other is swinging, the energy is actually transferred in its entirety, i.e. one pendulum always comes to rest while the other is swinging at its maximum amplitude. The time between two such occurrences of rest for one pendulum or, more generally, the time between any two instances of minimum amplitude is referred to as the beat period T_Δ.

The oscillation of two identical coupled ideal pendulums can be regarded as a superimposition of two natural oscillations. These natural oscillations can be observed when both pendulums are fully in phase or fully out of phase. In the first case, both pendulums vibrate at the frequency that they would if the coupling to the other pendulum were not present at all. In the second case, the effect of the coupling is at a maximum and the inherent frequency is greater. All other oscillations can be described by superimposing these two natural oscillations.

The equation of motion for the pendulums takes the form:

(1)
$$L\cdot\varphi_1+g\cdot\varphi_1+k\cdot(\varphi_1-\varphi_2)=0$$
$$L\cdot\varphi_2+g\cdot\varphi_2+k\cdot(\varphi_2-\varphi_1)=0$$

g: Acceleration due to gravity, L: length of pendulum, k: coupling constant

For the motions $\varphi_-=\varphi_1-\varphi_2$ and $\varphi_+=\varphi_1+\varphi_2$

(initially chosen arbitrarily) the equation of motion is as follows:

(2)
$$L\cdot\varphi_++g\cdot\varphi_+=0$$
$$L\cdot\varphi_-+(g+2k)\cdot\varphi_-=0$$

The solutions

(3)
$$\varphi_+=a_+\cdot\cos(\omega_+t)+b_+\cdot\sin(\omega_+t)$$
$$\varphi_-=a_-\cdot\cos(\omega_-t)+b_-\cdot\sin(\omega_-t)$$

give rise to angular frequencies

(4)
$$\omega_+=\sqrt{\frac{g}{L}}$$
$$\omega_-=\sqrt{\frac{g+2k}{L}}$$

corresponding to the natural frequencies for in phase or out of phase motion ($\varphi_+=0$ for out of phase motion and $\varphi_-=0$ for in-phase motion).

The deflection of the pendulums can be calculated from the sum or the difference of the two motions, leading to the solutions

(5)
$$\varphi_1=\frac{1}{2}\cdot\left(a_+\cdot\cos(\omega_+t)+b_+\cdot\sin(\omega_+t)+a_-\cdot\cos(\omega_-t)+b_-\cdot\sin(\omega_-t)\right)$$
$$\varphi_2=\frac{1}{2}\cdot\left(a_+\cdot\cos(\omega_+t)+b_+\cdot\sin(\omega_+t)-a_-\cdot\cos(\omega_-t)-b_-\cdot\sin(\omega_-t)\right)$$

Parameters a_+, a_-, b_+ and b_- are arbitrary coefficients that can be calculated from the initial conditions for the two pendulums at time $t = 0$.

It is easiest to consider the following case where pendulum 1 is moved at time 0 from rest to an initial angular velocity ψ_0 while pendulum 2 remains at rest.

(6)
$$\varphi_1=\frac{1}{2}\cdot\left(\frac{\psi_0}{\omega_+}\cdot\sin(\omega_+t)+\frac{\psi_0}{\omega_-}\cdot\sin(\omega_-t)\right)$$
$$\varphi_2=\frac{1}{2}\cdot\left(\frac{\psi_0}{\omega_+}\cdot\sin(\omega_+t)-\frac{\psi_0}{\omega_-}\cdot\sin(\omega_-t)\right)$$

The speed of both pendulums is then given by:

(7)
$$\varphi_1=\frac{\psi_0}{2}\cdot\left(\cos(\omega_+t)+\cos(\omega_-t)\right)$$
$$\varphi_2=\frac{\psi_0}{2}\cdot\left(\cos(\omega_+t)-\cos(\omega_-t)\right)$$

which can be rearranged to give

(8)
$$\varphi_1=\psi_0\cdot\cos(\omega_\Delta t)\cdot\cos(\omega t)$$
$$\varphi_2=\psi_0\cdot\sin(\omega_\Delta t)\cdot\cos(\omega t)$$

where (9)
$$\omega_\Delta=\frac{\omega_--\omega_+}{2}$$
$$\omega=\frac{\omega_++\omega_-}{2}$$

This corresponds to an oscillation of both pendulums at identical angular frequency ω, where the velocity amplitudes ψ_1 and ψ_2 are modulated at an angular frequency ω_Δ:

(10)
$$\psi_1(t)=\psi_0\cdot\cos(\omega_\Delta t)$$
$$\psi_2(t)=\psi_0\cdot\sin(\omega_\Delta t)$$

EVALUATION

Equation (4) can be used to calculate the natural oscillation periods T_+ and T_- for in-phase and out-of-phase oscillation:

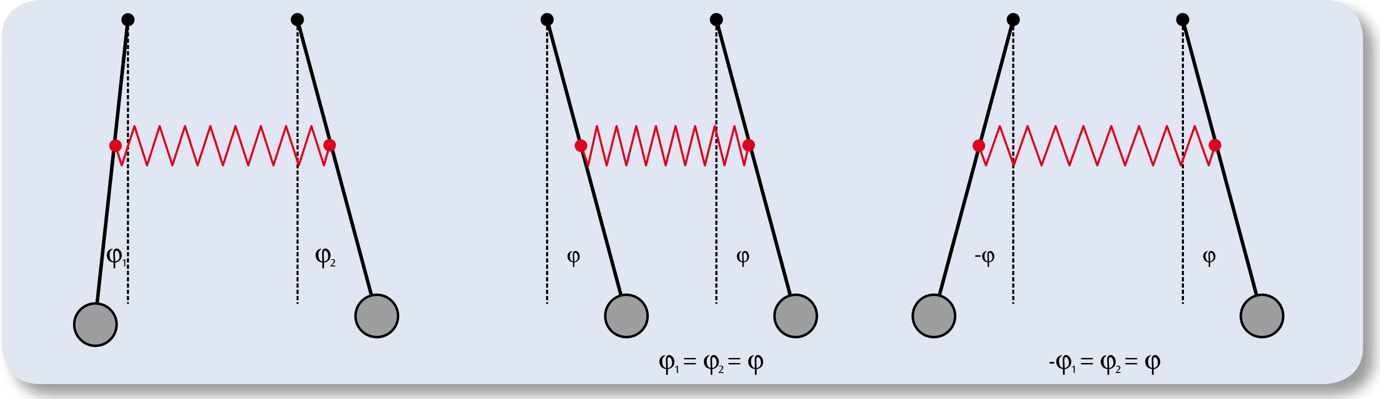
$$T_+=\frac{2\pi}{\omega_+}=2\pi\sqrt{\frac{L}{g}}\quad\text{and}\quad T_-=\frac{2\pi}{\omega_-}=2\pi\sqrt{\frac{L}{g+2k}}$$

For a period T for coupled oscillation, equation (9) implies the following:

$$\frac{2\pi}{T}=\omega=\frac{\pi}{T_+}+\frac{\pi}{T_-}\quad\text{and therefore}\quad T=2\cdot\frac{T_+\cdot T_-}{T_-+T_+}$$

The amplitude modulation given in equation (10) is usually stipulated in terms of its period T_Δ corresponding to the time between successive points where one pendulum stands still:

$$\frac{2\pi}{2T_\Delta}=\omega_\Delta=\frac{\pi}{T_-}-\frac{\pi}{T_+}\quad\text{and therefore}\quad T_\Delta=\frac{T_+\cdot T_-}{T_+-T_-}$$



General coupled oscillation | Coupled oscillation in phase | Coupled oscillation out of phase