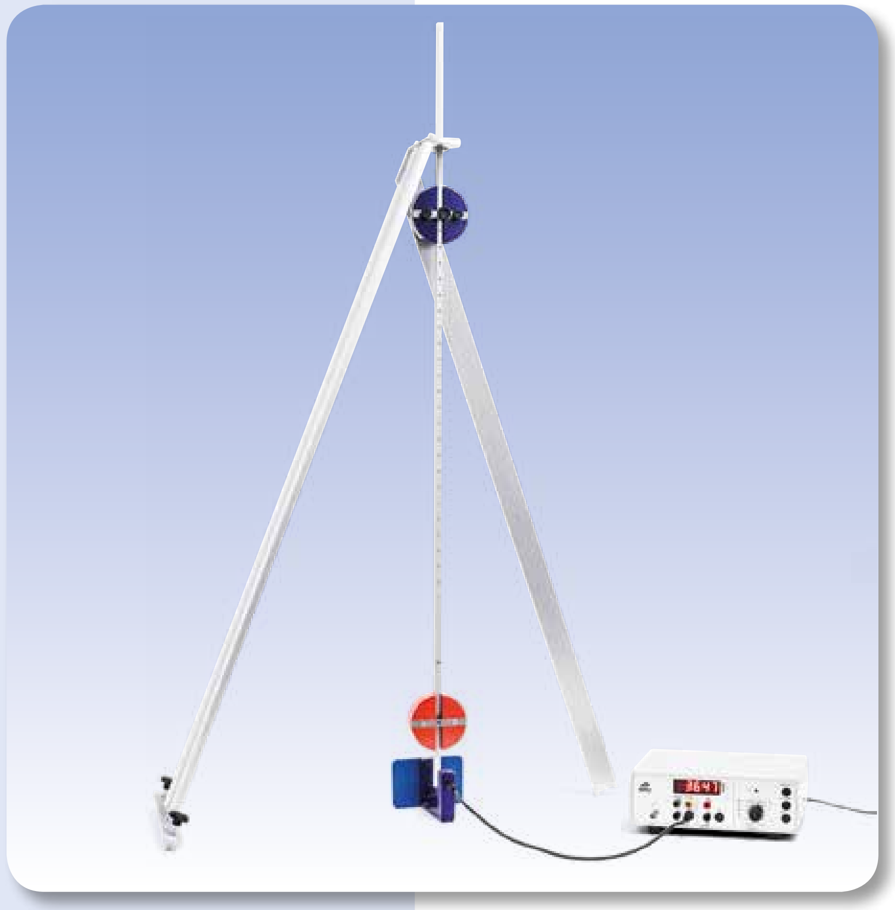
UE1050221 KATER'S REVERSIBLE PENDULUM



OBJECTIVE

Work out the local acceleration due to gravity with the help of a reversible pendulum

EXPERIMENT PROCEDURE

- **Configure a reversible pendulum such that the periods of oscillation are the same from both mounting points.**
- **Determine the period of oscillation and calculate the local acceleration due to gravity.**

SUMMARY

A reversible pendulum is a special design of a normal physical pendulum. It is able to swing from either of two mounting points and can be set up in such a way that the period of oscillation is the same from both these points. The reduction in the length of the pendulum then matches the distance between the two mounting points. This makes it easier to determine the local acceleration due to gravity from the period of oscillation and the reduced pendulum length. Matching of the reversing pendulum is achieved by moving a weight between the mounts as appropriate while a rather larger counterweight outside that length remains fixed.

REQUIRED APPARATUS

Quantity	Description	Number
1	Kater's Reversible Pendulum	1018466
1	Photo Gate	1000563
1	Digital Counter (230 V, 50/60 Hz)	1001033 or
	Digital Counter (115 V, 50/60 Hz)	1001032

You can find technical information about the equipment at 3bscientific.com

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BASIC PRINCIPLES

A reversible pendulum is a special design of a normal physical pendulum. It is able to swing from either of two mounting points and can be set up in such a way that the period of oscillation is the same from both these points. The reduction in the length of the pendulum then matches the distance between the two mounting points. This makes it easier to determine the local acceleration due to gravity from the period of oscillation and the reduced pendulum length.

If a physical pendulum oscillates freely about its rest position with a small deflection ϕ then its equation of motion is as follows:

(1) $$\frac{J}{m \cdot s} \cdot \ddot{\varphi} + g \cdot \varphi = 0.$$

J: Moment of inertia about axis of oscillation,
g: Acceleration due to gravity, m: Mass of pendulum,
s: Distance between axis of oscillation and centre of gravity

The reduced length of the physical pendulum is

(2) $$L = \frac{J}{m \cdot s}$$

A mathematical pendulum of this length oscillates with the same period of oscillation.

Steiner's law gives us the moment of inertia:

(3) $$J = J_s + m \cdot s^2.$$

J_S: Moment of inertia about centre of gravity axis

For a reversible pendulum with two mounting points separated by a distance d, the reduced lengths to be assigned are therefore

(4) $$L_1 = \frac{J_S}{m \cdot s} + s \quad \text{and} \quad L_2 = \frac{J_S}{m \cdot (d - s)} + d - s$$

They match up if the reversible pendulum is configured in such a way that the period of oscillation is the same for both mounting points. In that case, the following is true:

(5) $$s = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 - \frac{J_S}{m}}$$

and

(6) $$L_1 = L_2 = d.$$

In this case, the period of oscillation T is given by

(7) $$T = 2\pi \cdot \sqrt{\frac{d}{g}}.$$

In the experiment, matching of the reversible pendulum is accomplished by moving a weight of mass m_2 = 1 kg between the mounting points as appropriate. A second large counterweight of mass m_1 = 1.4 kg is fixed outside the mounts. Measurement of the period of oscillation is handled electronically with the lower end of the pendulum periodically interrupting a photoelectric gate. By this means, the periods of oscillation T_1 and T_2 associated with the reduced pendulum lengths L_1 and L_2 are measured as a function of the position x_2 of weight m_2.

EVALUATION

The two curves derived from the measurements $T_1(x_2)$ and $T_2(x_2)$ intersect twice at the value $T = T_1 = T_2$. To determine the intersects accurately requires interpolation between the measurement points themselves. Acceleration due to gravity is calculated from the measurements as follows:

$$g = \left(\frac{2\pi}{T}\right)^2 \cdot d, \; d = 0.8 \text{ m}$$

with relative precision of 0.3 per thousand.

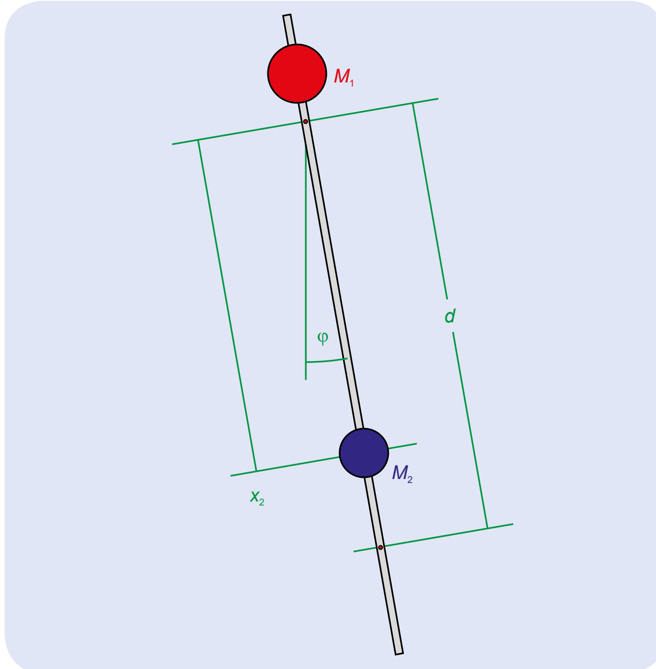

Fig. 1: Schematic diagram of a reversible pendulum

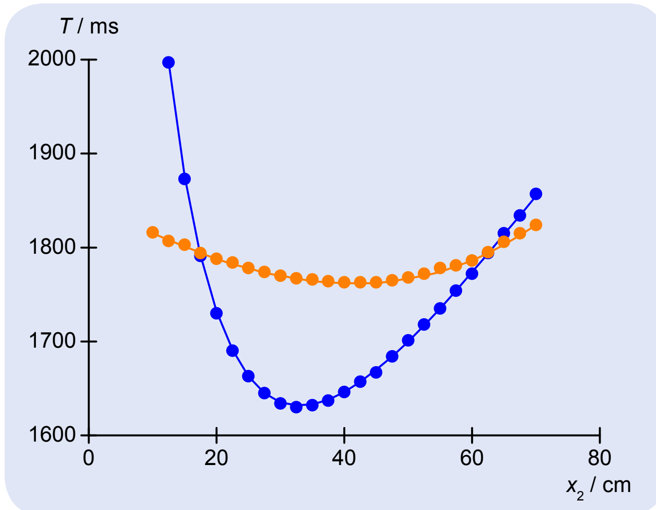

Fig. 2: Measured periods of oscillation T_1 und T_2 as a function of position of weight 2.