PARALLELOGRAM OF FORCES

OBJECTIVE

Experimental investigation of the vector addition of forces.

SUMMARY

The vector addition of forces can be demonstrated in a clear and simple manner on the force table. The point of action of three individual forces in equilibrium is exactly in the middle of the table. Determine the magnitude of the individual forces from the suspended weights and, using a protractor, note the angle of each force vector (the direction of each force). The result of the experiment can be evaluated analytically or represented as a graph.

EXPERIMENT PROCEDURE

- **Plotting the equilibrium of three arbitrary forces on a graph**
- **Analytical investigation of the point of equilibrium when forces F_1 and F_2 are symmetrical**

REQUIRED APPARATUS

Quantity	Description	Number
1	Force Table	**1000694**

BASIC PRINCIPLES

Forces are vectors and can therefore be added using the rules of vector addition. To demonstrate the sum of two vectors on a graph, the point of origin of the second vector is placed on the final point of the first vector. The arrow from the point of origin of the first vector to the final point of the second vector represents the resultant vector. By completing the parallelogram (of which the two vector lines are sides), a diagonal drawn from the original angle to the opposite corner represents the resultant vector (also see Fig. 1).

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In a state of equilibrium, the sum of the three individual forces is given by:

(1) $$F_1 + F_2 + F_3 = 0$$

$-F_3$ is therefore the sum of individual forces F_1 and F_2 (also see Fig. 2):

(2) $$-F_3 = F = F_1 + F_2$$

The parallel vector components for sum F are given by

(3) $$-F_3 = F = F_1 \cdot \cos\alpha_1 + F_2 \cdot \cos\alpha_2$$

and the vertical components are given by

(4) $$0 = F_1 \cdot \sin\alpha_1 + F_2 \cdot \sin\alpha_2$$

Equations (3) and (4) provide a mathematical analysis of the vector addition. For the experiment, it is advisable to align force F_3 at an angle of 0°.

For analytical observations, the equilibrium of forces can alternatively be investigated on a graph. To do so, draw lines representing all three forces diverging from the central point of action. Note the magnitude and angle of each force. Subsequently, displace forces F_2 and F_3 along a parallel path till the point of origin is at the end of the preceding vector. The resultant vector is 0 (also see Fig. 3). In the experiment, carry out this procedure for three arbitrary forces, making sure to maintain the state of equilibrium every time.

In the experiment, the analytical observation is restricted to the special situation that the two forces F_1 and F_2 are symmetric to F_3.

EVALUATION

Equation (4) is satisfied in a symmetric case ($F_1 = F_2$ and $\alpha_1 = -\alpha_2$. From equation (3) we get the characteristic equation applied in Fig. 4 (for describing the measurement data).

$$F = 2 \cdot F_1 \cdot \cos\alpha_1$$

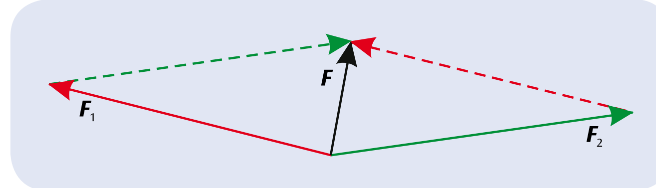

Fig. 1: Vector sum of forces (parallelogram of forces).

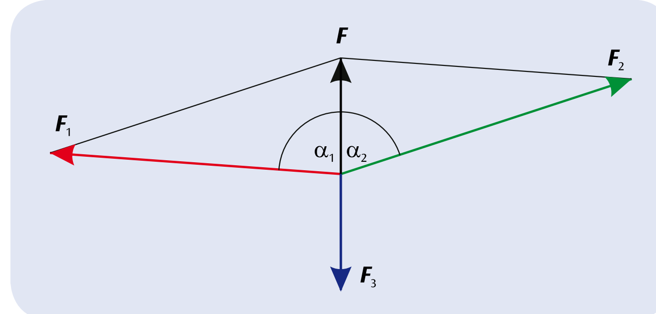

Fig. 2: Determining the sum of vectors of two forces F_1 and F_2 from equilibrium force F_3.

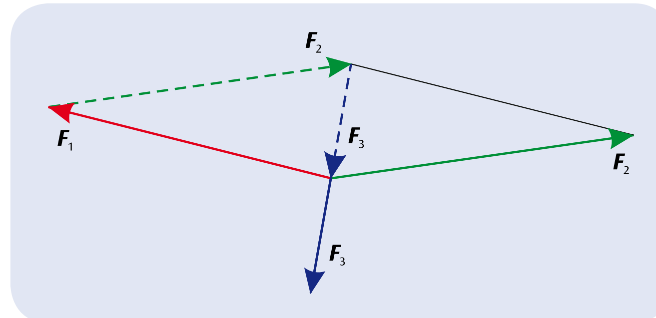

Fig. 3: Graphic investigation of the equilibrium of three arbitrary forces acting in different directions.

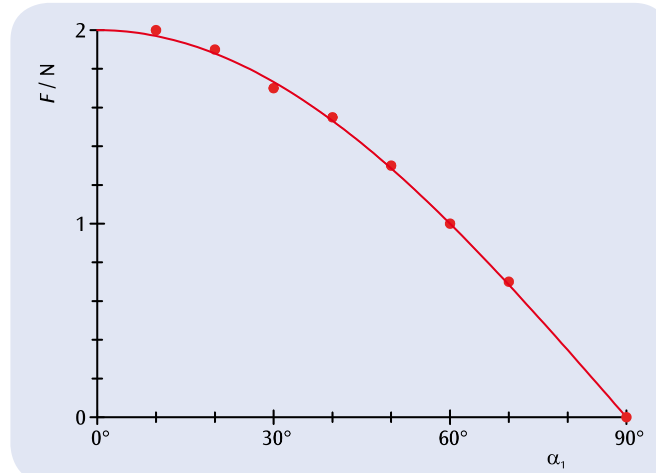

Fig. 4: Measured and calculated sums of two symmetric forces in relation to the angle α_1.