

**OBJECTIVE**

Confirm the law of equal areas for central force motions (Kepler's Second Law).

SUMMARY

As an example of motion under the influence of a central force, the elliptical motion of a pendulum bob is recorded by the dust-marking method. This produces a trace with time-interval marks, and from the distances between these one can directly measure the velocity of the pendulum bob. Furthermore, it can be shown by a simple graphical analysis that the area swept by the radius vector of the pendulum in each time interval is constant, and is thus independent of the length of the radius vector.

EXPERIMENT PROCEDURE

- Recording the elliptical oscillations of a pendulum by the dust-marking method.
- Comparing the velocities of the pendulum bob at the minimum and maximum distances from its rest position.
- Measuring the areas swept by the radius vector of the pendulum in each time interval at the minimum and maximum distances from the rest position.

REQUIRED APPARATUS

Quantity	Description	Number
1	Equipment Set for Powder Tracing	1000739
1	Pendulum with Plotting Electrode	1000780
2	Tripod Stands 150 mm	1002835
2	Stainless Steel Rods 1000 mm	1002936
1	Stainless Steel Rod 750 mm	1002935
3	Multiclamps	1002830

1

BASIC PRINCIPLES

In the motion of a planet around the sun, the angular momentum remains constant, because the force acting on the planet is always directed towards the centre. From this it can be directly concluded that the planet's orbit must lie in a fixed plane. From it one can also derive Kepler's Second Law, the law of equal areas, which states that the light-beam from the sun to the planet sweeps equal areas in equal time intervals.

The validity of the law of equal areas is not affected by the exact form of the dependence of the central force on the distance from the force centre. This dependence only determines the shape of the orbit around the force centre. Thus, the equal areas law is also valid for the elliptical oscillations of a pendulum around the rest position, provided that the deflection angle from the vertical is not too great. The movement of the pendulum bob is almost exactly confined to a horizontal plane (see Fig. 1), and at any point on its path defined by the radius vector r there is a horizontal restraining force F directed towards the rest position, given by:

$$(1) \quad F = -\frac{m \cdot g}{d} \cdot r$$

g : gravitational acceleration,
 d : length of pendulum,
 m : mass of pendulum bob.

The angular momentum, which is

$$(2) \quad L = m \cdot r(t) \times \frac{\Delta r(t)}{\Delta t}$$

remains unaffected by force F . Therefore, the area ΔA swept by the radius vector $r(t)$ in each time interval Δt also remains constant:

$$(3) \quad \Delta A = \frac{1}{2} |r(t) \times \Delta r(t)| = \frac{1}{2} \cdot r(t) \cdot \Delta r(t) \cdot \sin \alpha$$

(see Fig. 2).

In the experiment the motion of the pendulum bob is recorded by the dust-marking method. In this method, the recording electrode attached to the pendulum bob is allowed to glide above an insulated tracking plate covered with fine sulphur powder. An alternating voltage at the mains frequency is applied between the electrode and the tracking plate, so that the sulphur powder is alternately attracted and repelled according to the changing polarity. This draws a trace consisting of time marks, and from the distances between these one can directly determine the velocity of the pendulum bob.

EVALUATION

At first, plot a graphical representation and determine the centre of the recorded path and the points on the orbit that correspond to the maximum and minimum distances from the centre.

At each of these points on the orbit, it is possible to determine the area swept by the radius vector in 10 cycles of the alternating voltage. For purposes of simplicity, the areas are calculated by treating them approximately as triangles.

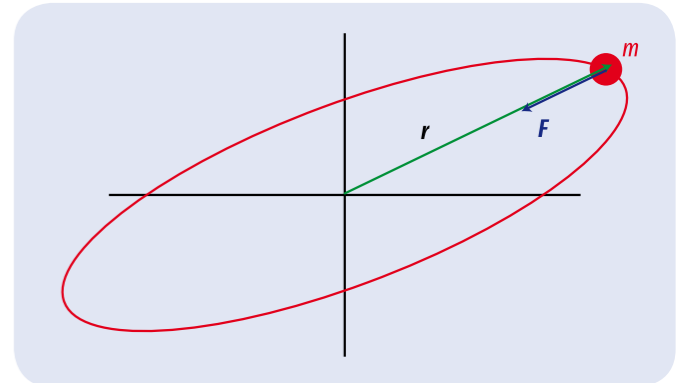


Fig. 1: Elliptical oscillation of the pendulum bob viewed from above

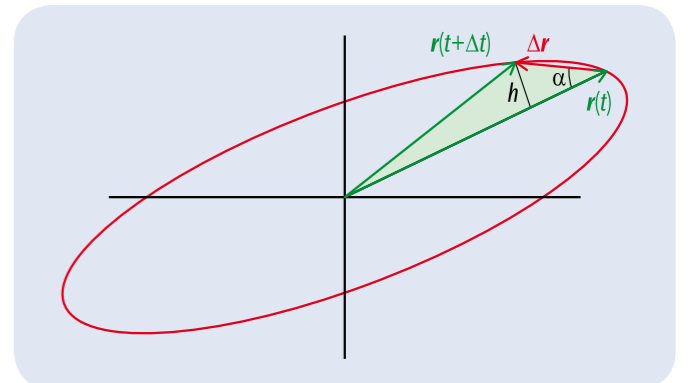


Fig. 2: The area swept by the radius vector of the pendulum bob in the time interval Δt

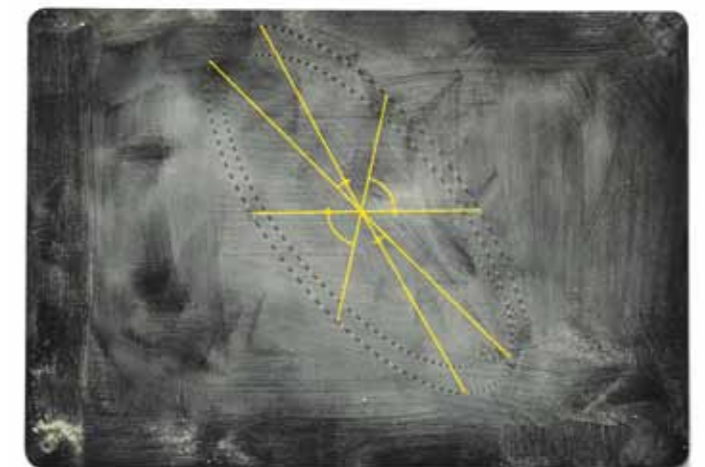


Fig. 3: Example of experimentally measured data with calculation